



University of Ibn Khaldoun –Tiaret
Faculty of Materials Science
Physics Department

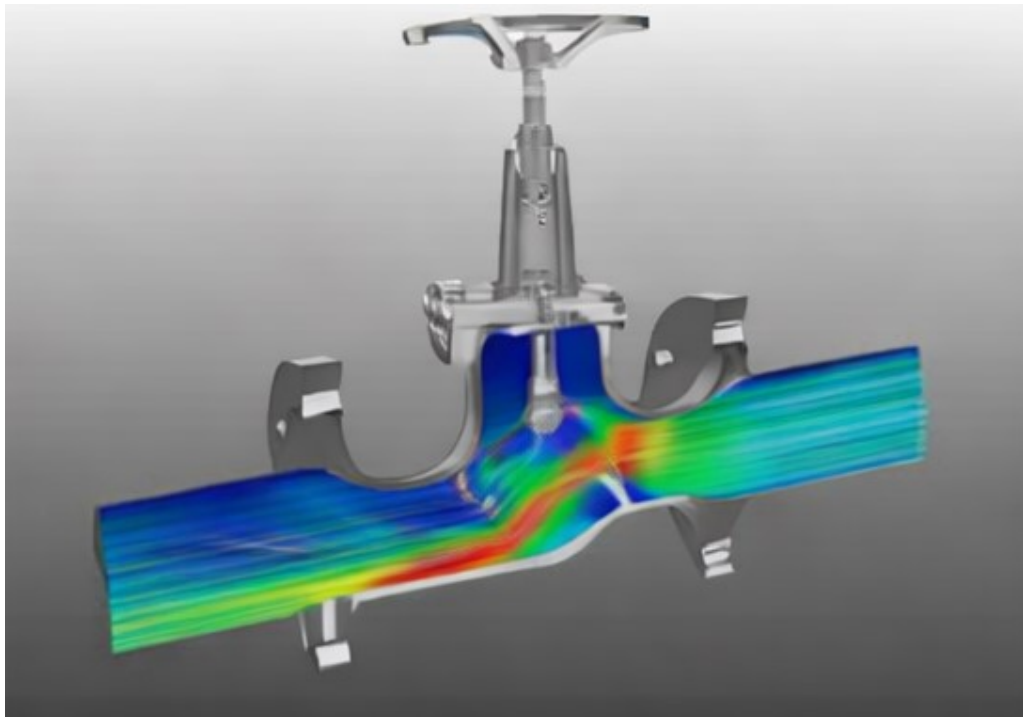


Fluid Mechanics II

Lecture and Tutorials

« Made for energetic physics and mechanics learners »

Author : Dr. TRARI Benaissa



2025/2026

CONTENTS

INTRODUCTION.....	6
i. Foreword	7
ii. Course Positioning within the Curriculum.....	7
iii. Importance of Fluid Mechanics in Engineering Sciences	7
iv. Links with Related Disciplines	7
v. General Organization of the Course Notes	8
vi. Course Information	8
Chapter I. REVIEW OF FLUID MECHANICS	9
I.1. Learning Objectives.....	10
I.2. Context and importance	10
I.3. Introduction	11
I.4. The continuous approach's value.....	12
I.3.1. Mechanics branches.....	12
I.3.2. Fluid mechanics branches.....	13
I.3.3. Fluid	13
I.4. Continuum hypothesis.....	15
I.5. Characteristics of fluid medium.....	15
I.5.1. Viscosity.....	16
I.5.2. Dynamic viscosity	16
I.5.3. Kinematic Viscosity	17
I.5.4. Newton's formula for Coefficient of Viscosity	17
I.5.5. Thermal conductivity k	18
I.5.6. Mass diffusivity.....	18
I.6. Body Forces vs surface forces	19
I.7. Ideal fluid vs Real fluid	19
Chapter II. FLUID STATICS	21
II.1. Learning Objectives.....	22
II.2. Introduction	22
II.3. Fluid statics	23
II.4. Pressure	23
II.4.1. Unit of pressure	23
II.4.2. Measurement of Pressure	23
II.4.3. Characteristics of pressure	24
II.4.4. Variation of Pressure with Depth	25

II.5.	Fundamental Principal of Hydrostatic.....	26
II.5.1.	Pascal's law	26
II.6.	Manometers.....	27
II.6.1.	Barometer	27
II.6.2.	Manometer.....	28
II.7.	Buoyancy.....	29
II.7.1.	Archimedes' Principle	29
II.7.2.	Center of gravity and buoyancy.....	31
II.7.3.	Stability of Immersed Bodies.....	31
II.8.	Problems Set.....	33
Chapter III.	FLUID KINEMATICS.....	38
III.1.	Prerequisite Knowledge	39
III.2.	Introduction	39
III.3.	Description of Fluid Motion.....	39
III.3.1.	Velocity field.....	39
III.4.	Lagrangian and Eulerian Description	40
III.4.1.	Lagrangian approach.....	40
III.4.2.	Concept of path line.....	41
III.4.3.	Eulerian approach.....	41
III.4.4.	Concept of streamline.....	41
III.5.	Advantages of Eulerian approach	42
III.6.	Path line and Streamline equation.....	42
III.6.1.	Path line equation	42
III.6.2.	Stream line Equation.....	43
III.7.	Deformation of Fluid Elements	43
III.7.1.	Translation.....	43
III.7.2	Rotation (vorticity).....	43
III.7.3.	Deformation (strain rates)	44
III.7.4.	Shear Stress vs Shear Strain Rate.....	44
III.8.	Vorticity and Irrotationality.....	45
III.9.	Flows Properties	46
III.9.1.	Planar flows	46
III.9.2.	Steady Flow	46
III.9.3.	Conservative flow (without fluid production).....	46
III.9.4.	incompressible fluid	47
III.9.5.	Acceleration Field.....	47

III.10.	Flow Visualization & Measurement	48
III.10.1.	Types of Flow Visualization Techniques.....	48
III.10.2.	Application Areas.....	50
Chapter IV.	PERFECT FLUID DYNAMICS	56
IV.1.	Prerequisite Knowledge.....	57
IV.2.	Learning Objectives.....	57
IV.3.	Introduction & Context.....	57
IV.4.	Definition of Incompressible Perfect Fluid	58
IV.5.	Conservation of Mass	58
IV.6.	Mass and Volume Flow Rates	58
IV.6.1.	Mass Flow Rate.....	58
IV.6.2.	Volume flow rate	59
IV.7.	Continuity equation.....	59
IV.8.	Newton's Second Law	60
IV.9.	Bernoulli Theorem	63
IV.9.1.	Energy Conservation and Bernoulli's Equation.....	64
IV.9.2.	Bernoulli's Equation.....	65
IV.10.	hydraulic head	67
IV.10.1.	Bernoulli's principle and Venturi effect.....	67
IV.10.2.	Applications of the Venturi Effect.....	68
IV.11.	Static, Stagnation, Dynamic, and Total Pressure	71
IV.11.1.	Static Pressure.....	71
IV.11.2.	Dynamic pressure.....	71
IV.11.3.	Stagnation pressure (total pressure).....	71
IV.12.	Problem Set.....	72
Chapter V.	REAL FLUID DYNAMICS.....	78
V.1.	Learning Objectives.....	79
V.2.	Prerequisite Knowledge.....	79
V.2.1.	Mathematics Prerequisites	79
V.2.2.	Physics Prerequisites	79
V.3.	Introduction & Context.....	79
V.4.	Definition.....	80
V.5.	Ideal Fluid vs. Real Fluid: Key Differences	80
V.5.1.	What is an Ideal Fluid?.....	80
V.5.2.	What is a Real Fluid?	81
V.6.	Viscosity.....	82
V.6.1.	Dynamic viscosity.....	82

V.6.2.	Kinematic viscosity.....	83
V.7.	Types of flow.....	83
V.7.1.	Laminar Flow	83
V.7.2.	Turbulent Flow	84
V.7.3.	Reynolds Number and Flow Pattern.....	84
V.8.	Hydraulic Diameter.....	85
V.9.	Pressure Drop	86
V.10.	Moody Diagram	88
V.11.	Head loss	89
V.11.1.	Major Loss (regular loss)	90
V.11.2.	Minor loss (singular loss).....	91
V.12.	Bernoulli's theorem applied to a real fluid.....	92
V.12.1.	Energy form of Bernoulli equation.....	92
V.12.2.	Pressure form of Bernoulli equation	92
V.12.3.	Head form of Bernoulli equation.....	93
V.13.	Poiseuille's law	93
V.14.	Problems Set	95
VI.	APPENDICES.....	100
VI.1.	Appendices	101
VI.1.1.	Mathematical reminders	101
VI.1.2.	Fluid property tables	103
VI.1.3.	Physical Constants.....	104
	Bibliography.....	105
	Electronic web site.....	112

INTRODUCTION

i. Foreword

The mechanics of incompressible fluids is a fundamental discipline in energy. It provides the theoretical and methodological foundations necessary for the analysis and understanding of fluid phenomena, whether the fluids are at rest or in motion.

This handout has been prepared to serve as a teaching aid for students of physics and energy mechanics and to facilitate the progressive acquisition of the essential concepts of this discipline.

ii. Course Positioning within the Curriculum

This course is part of the Bachelor's or Master's degree program in Physical Sciences and Mechanics, Civil Engineering, and related disciplines. It follows courses in Mathematics, General Physics, and Thermodynamics, and provides an essential foundation for the study of specialized modules. Mastery of the concepts presented in this course material is required for subsequent advanced courses in compressible fluid mechanics, particularly aerodynamics.

iii. Importance of Fluid Mechanics in Engineering Sciences

Incompressible fluid mechanics plays a central role in the training of undergraduate and graduate students in engineering due to its broad range of applications. It forms the basis for the study of liquid flow in pipes, channels, hydraulic machines, and industrial installations. It allows for the analysis of pressure, flow rate, head loss, and the forces exerted by fluids on structures. The fundamental principles derived from it are essential for the design, operation, and optimization of hydraulic, energy, and environmental systems.

iv. Links with Related Disciplines

Incompressible fluid mechanics is closely linked to several disciplines within energy sciences. It interacts with thermodynamics in the study of energy balances of constant-density flows and forms a foundation for heat transfer by forced or natural convection. It also plays a major role in process engineering, particularly in the design of fluid transport and mixing equipment. In biomechanics, it allows for the modeling of blood flow with low compressibility, while in hydraulics and civil engineering, it is essential for the analysis of pressurized and free-surface flows. These interactions highlight the cross-disciplinary nature of incompressible fluid mechanics in energy studies.

v. General Organization of the Course Notes

The course notes are structured progressively to facilitate the assimilation of the fundamental concepts of incompressible fluid mechanics.

It begins with a chapter devoted to general principles and the physical properties of fluids, followed by the study of the statics of incompressible fluids. The following chapters address the dynamics of incompressible perfect fluids, then those of incompressible viscous fluids, emphasizing conservation laws, flow regimes, and practical applications.

Each chapter includes theoretical explanations, illustrative examples, and application exercises that connect the fundamental concepts to concrete situations encountered in engineering. In addition, a prerequisite test is provided at the beginning of each chapter to assess the knowledge required for its study, while a final self-assessment test is available at the end of the chapter.

These assessments are posted online on the Moodle learning platform "<https://moodle.univ-tiaret.dz/my/courses.php>", promoting progressive, interactive and autonomous learning, as well as continuous monitoring of student results.

vi. Course Information

Faculty: Materials Science

Department: Physics

Target audience: Physics and Mechanics at the Bachelor's and Master's Levels

Course Title: Fluid Mechanics II

Teaching unit: UEF1

Module: F512 Fluid Mechanics II

Credit: 06

Coefficient: 03

Duration: The course is divided into 2 lectures and 1 classroom activity per week

Teacher: Dr. TRARI Benaissa

Contact mail: b_trari@univ-tiaret.dz

Chapter I. REVIEW OF FLUID MECHANICS

I.1. Learning Objectives

At the end of this chapter, students should be able to:

- a. Define **fluid mechanics** and explain its importance in science and engineering applications.
- b. Define a **fluid** and distinguish it from a solid based on its response to **shear stress**.
- c. Explain the **continuum hypothesis** and discuss its role in the **mathematical modeling** of fluid flows.
- d. Identify and describe the main physical **properties of fluids**, including:
 - o density
 - o viscosity
 - o mass diffusivity
 - o thermal conductivity
- e. Interpret the physical significance of **viscosity**, and distinguish between Newtonian and non-Newtonian fluid behavior.
- f. Differentiate between **ideal** fluids and **real** fluids, and explain the influence of viscosity on fluid motion.
- g. Distinguish between **body forces** and **surface forces**, and identify common examples encountered in fluid systems.
- h. Classify fluid flows according to different physical criteria:
- i. Use appropriate fluid mechanics **terminology** and **notation** for the description of basic fluid phenomena.
- j. Develop a physical understanding of fluid **behavior** as a foundation for the study of fluid statics, kinematics, and dynamics.

I.2. Context and importance

This first chapter provides an essential introduction to the study of incompressible fluid mechanics. Its goal is to provide the theoretical and practical foundation needed to comprehend the fluid statics and dynamics chapters that follow. The theoretical structure that will be used to develop the basic laws governing fluid behavior is defined by the ideas discussed in this chapter.

The study of the physical properties of incompressible fluids, such as density, viscosity, and pressure, is essential for accurately describing and modeling flow phenomena. These quantities appear systematically in the fundamental equations of fluid mechanics and influence the choice of simplifying assumptions adopted in the analysis of real flows.

By providing a common foundation of definitions, concepts, and notations, this chapter plays a structuring role throughout the entire course. It harmonizes the scientific vocabulary used and prepares students for the rigorous analysis of statics and dynamics problems of incompressible fluids encountered in engineering and industrial applications.

1.3. Introduction

In this introductory chapter, the basic ideas needed for fluid flow analysis are covered. The theoretical and practical foundation for comprehending the behavior of fluids at rest and in motion is provided by fluid mechanics, a fundamental area of continuum mechanics.

A precise definition of a fluid and an analysis of its fundamental physical characteristics are given at the start of the chapter. The properties that set fluids apart from solids are highlighted, particularly their capacity to experience continuous deformation when subjected to shear stress. The vast range of flow behaviors seen in both natural and engineering systems are governed by these intrinsic characteristics.

A central concept developed in this chapter is "viscosity" which is a key idea that measures a fluid's internal resistance to deformation. There is a thorough discussion of the physical causes of viscosity as well as the factors that affect it, including temperature and fluid composition. Comprehending viscosity is crucial for differentiating between various fluid behaviors and simulating actual flows.

The chapter also introduces the basic categorization of fluid flows. Fluid motion can be divided into a number of complementary categories from a physical and engineering standpoint, such as:

- 💧 Depending on whether the fluid is moving or at rest, there are two types of flows: static and dynamic;
- 💧 The degree of flow organization and mixing determines whether a flow is laminar or turbulent.
- 💧 Depending on time dependence, flows can be either steady or unsteady;
- 💧 Compressible versus incompressible flows based on changes in density;
- 💧 Internal versus external flows, based on whether confining boundaries are present;

These classifications offer an organized framework for evaluating fluid behavior and choosing suitable modeling techniques. Additionally, the distinction between "surface forces" (such as pressure and viscous stresses, acting on fluid boundaries) and "body forces" (such as gravity, acting throughout the fluid volume) is introduced. The formulation of the governing equations of fluid motion depends on this distinction.

The chapter concludes by defining "real viscous fluids" and "ideal inviscid fluids" and introducing the idea of viscous and inviscid regions within a flow. Understanding the boundaries of theoretical models and simplifying complicated issues depend on this distinction.

The reader will have learned the fundamental conceptual knowledge needed to deal with more complex fluid mechanics topics by the end of this chapter, such as fluid statics, kinematics, and the dynamics of both ideal and viscous flows.

1.4. The continuous approach's value

In fluid mechanics, matter is considered continuous, that is to say, without discontinuities at the scale of the problem studied. This hypothesis makes it possible to apply the tools of differential and integral calculation([Anderson, 1992](#)).

The mathematical description is based on the concept of physical field (velocity, pressure, temperature, etc.), expressed by partial differential equations.

1.3.1. Mechanics branches

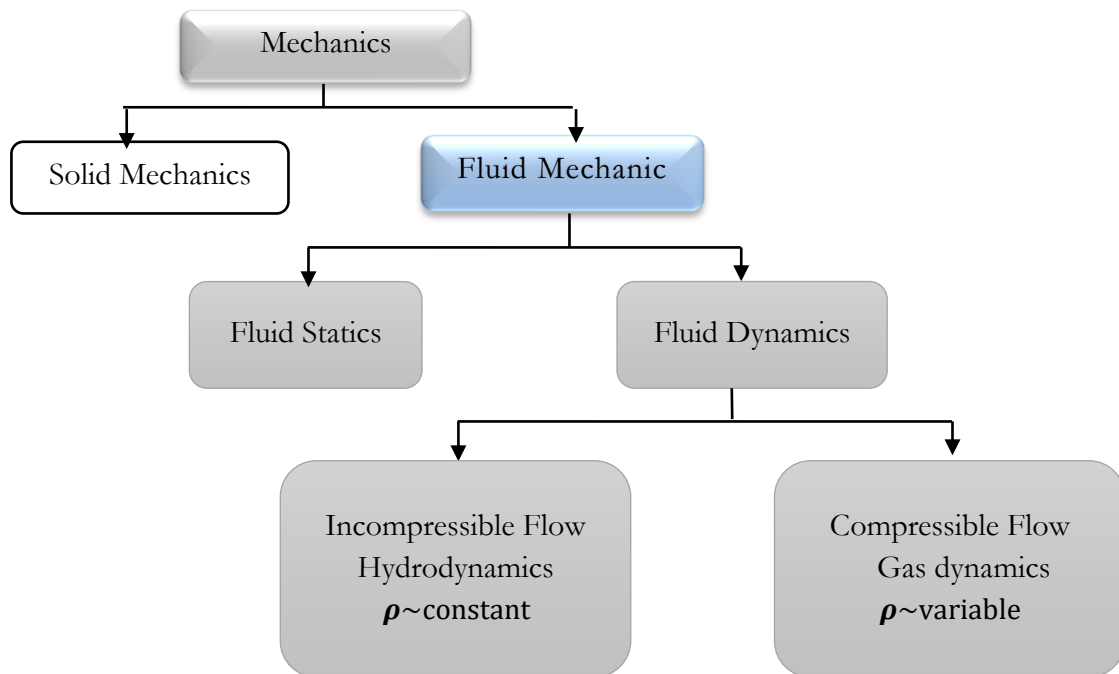


Fig.I.1 Mechanics Branches

1.3.2. Fluid mechanics branches

It is a branch of continuous mechanics which deals with the relationship between forces, motions, and statical conditions in continuous material (Genick Bar–Meir, 2013)

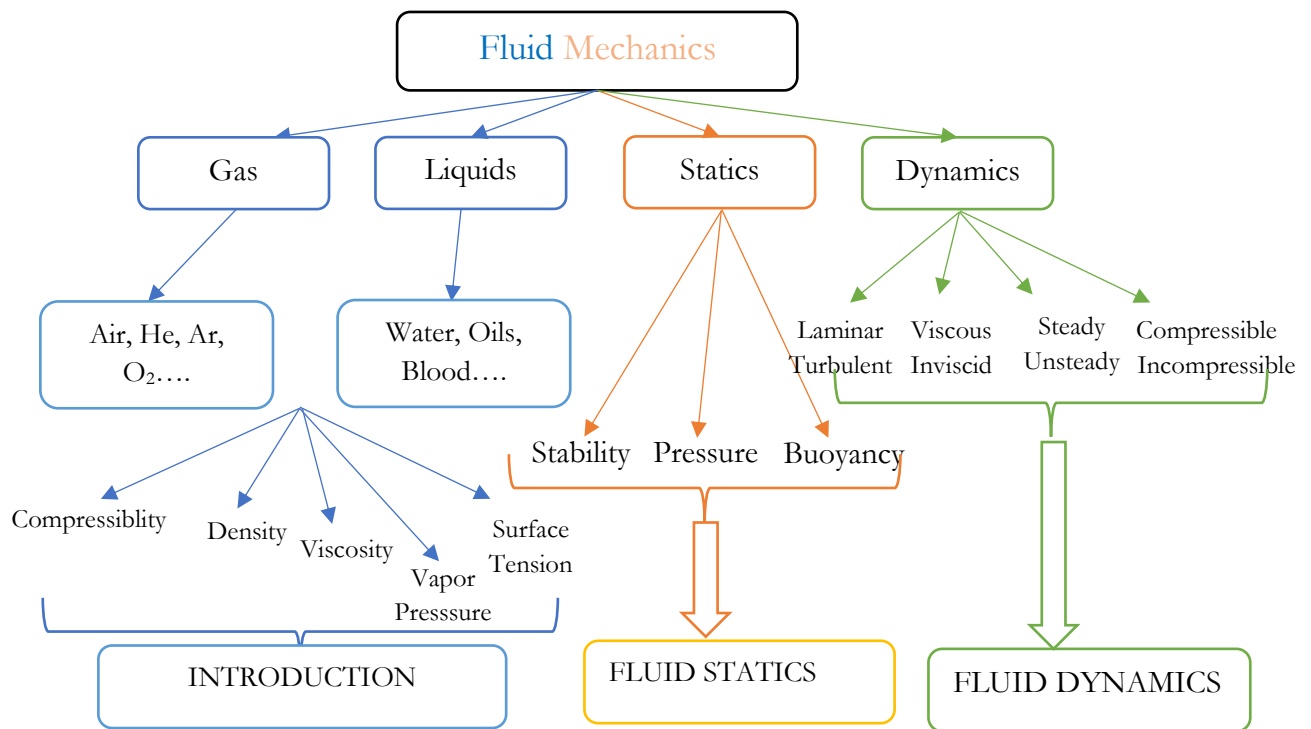


Fig.I.2 Fluid Mechanics branches (Song, 2018)

1.3.3. Fluid

The fluid is mainly divided into two categories: liquids and gases.

The main difference between the liquids and gases state is that gas can occupy the whole volume while liquids have an almost fix volume.

As shown in Figure 1.3, the liquid occupies the same volume in each container but changes its shape to conform to that of the container, forming an interface with liquid vapor above. The gas expands to completely fill the larger container—its volume is not fixed, and no interface is present.

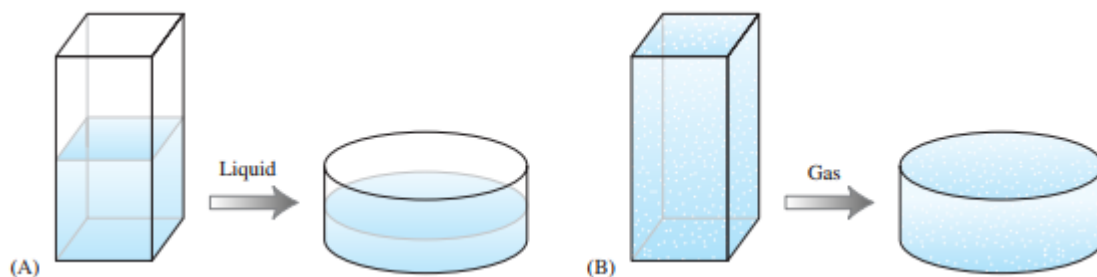


Fig.I.3 Differences in characteristics of liquids and gases (Poling et al., 2001).

(A) A liquid takes the shape of its container but retains a constant volume.

(B) A gas not only takes the shape of its container but expands or contracts to completely fill a container of any shape.

A fundamental distinction is also essential between gas and vapor ([Pascal, 2020](#)):

A gas is a fluid that remains in the gas phase under the usual conditions and can be modeled by the state equation of perfect gases.

A vapor, on the other hand, is a fluid close to the liquid phase, which can change state under the effect of variations in pressure or temperature.

Another distinction between liquids and gases is their compressibility, which main that the same weight on the piston compresses the gas into a substantially smaller volume than that occupied by the liquid under similar conditions. We conclude that all gases are far easier to compress than any liquid.

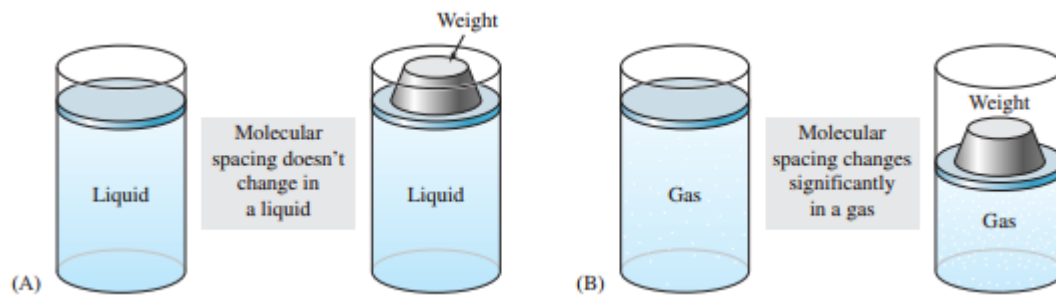


Fig.I.4 Differences in the compressibility of liquids and gases([Poling et al., 2001](#))

(A) The molecular spacing between liquid molecules is not changed appreciably when a weight is applied to the piston.

(B) In contrast, the same weight on the piston will cause a significant change (decrease) in the spacing between gas molecules.



Fig.I.5 Coffee foam is a mixture of gas bubbles in a liquid.

1.4. Continuum hypothesis

We are all familiar with fluids the most common being air and water and we experience them as being "smooth," i.e., as being a continuous medium. The concept of a continuum is the basis of classical fluid mechanics. The continuum assumption is valid in treating the behavior of fluids under normal conditions.

Fluid is composed of molecules which may be widely spaced apart, especially in the gas phase. As a consequence of the continuum assumption, each fluid property is assumed to have a definite value at every point in space. Thus, fluid properties such as density, temperature, velocity, and so on, are considered to be continuous functions of position and time.

Thus, fluid properties such as density, temperature, velocity, and so on, are considered to be continuous functions of position and time, defined as mass per unit volume. To determine the density at point, we must select a small volume, ΔV , surrounding the point and then determine the ratio $\Delta m / \Delta V$.

When the measuring volume ΔV is very small (approaching zero), the number of molecules in the volume will vary with time because of the random nature of molecular motion. (Elger et al., 2020)

The density at a "point" is then defined as:

$$\rho = \frac{\Delta m}{\Delta V} \quad (1.1)$$

The limiting volume ΔV is about 10^{-9} mm^3 for all liquids and for gases at atmospheric pressure. (Fox et al., 2020). Fluids are aggregations of molecules, widely spaced for a gas, closely spaced for a liquid (White, 2008).

The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space with no jump discontinuities (Cengel & Cimbala, 2017). Yet it is convenient to disregard the atomic nature of the fluid and view it as continuous, homogeneous matter with no holes, that is, a continuum.

1.5. Characteristics of fluid medium

Various properties of fluids which to some extent permit us to distinguish one fluid from another, and they allow us to make estimates of physical behavior of any specific fluid.

There are two main classes of properties to consider:

- 💧 Transport properties
- 💧 and (other, general) physical properties.

We will begin by considering three basic transport properties, namely viscosity, thermal conductivity and mass diffusivity (McDonough, 2009).

1.5.1. Viscosity

Most fluids offer some resistance to motion, and we call this resistance “viscosity.” Viscosity arises when there is relative motion between layers of the fluid.

More precisely, it measures resistance to flow arising due to the internal friction between the fluid layers as they slip past one another when fluid flows.

Viscosity can also be thought of as a measure of a fluid’s thickness or its resistance to objects passing through it.

The viscosity corresponds to the resistance of the fluid towards its moving. It is a response to a shear

constraint: $\frac{\vec{F}}{A}$, the ratio: $\frac{\vec{v}}{h}$ represents the speed gradient.

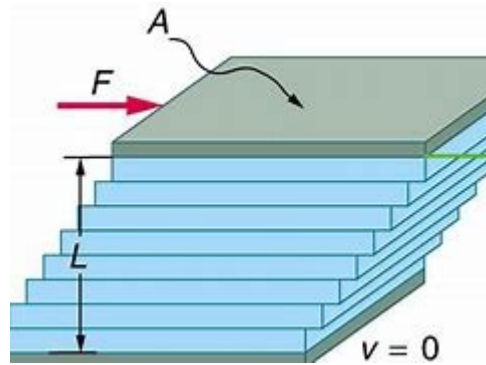


Fig.I.6 Schematics to describe the shear stress in fluid mechanics(MARCHESSE, 2019).

Represented by the symbol μ “eta”. where μ is a constant known as the coefficient of dynamic viscosity It is positive and is expressed in the international system in Poise .

$$\mu = \frac{\text{Shear stress}}{\text{Strain rate}} = \frac{f/A}{v/l} \quad (I.2)$$

It’s the ratio of the shearing stress (f/A) to the strain rate (dv/dx)

1.5.2. Dynamic viscosity

The proportionality factor η is the dynamic viscosity coefficient of fluid and the relationship between the slip force f and η is given by:

$$F = \mu \cdot S \frac{dv}{dx} \quad (I.3)$$

Observation: from the equation i.3, we conclude that “when the viscosity increases, the capacity of the fluid to flow decreases”.

1.5.3. Kinematic Viscosity

The cinematic viscosity coefficient is the relationship of dynamic viscosity on the density:

$$\nu = \eta / \rho \text{ [m}^2/\text{s]} \quad (\text{I.4})$$

Common unit used is the stoke (St) = [cm²/s]

SI unit of viscosity is the Pascal second [Pa. s], A common unit is the poise[P] where 1 Pa.s = 10P

1 Pa. s = 1 kg.m⁻¹. s⁻¹.

1.5.4. Newton's formula for Coefficient of Viscosity

Newton's law of viscosity states that "Shear stress (Tangential pressure) is directly proportional to Velocity gradient" Boundary plate (2D, moving) Velocity, U Shear stress (T) Gradient fluid Boundary plate (2D, stationary)

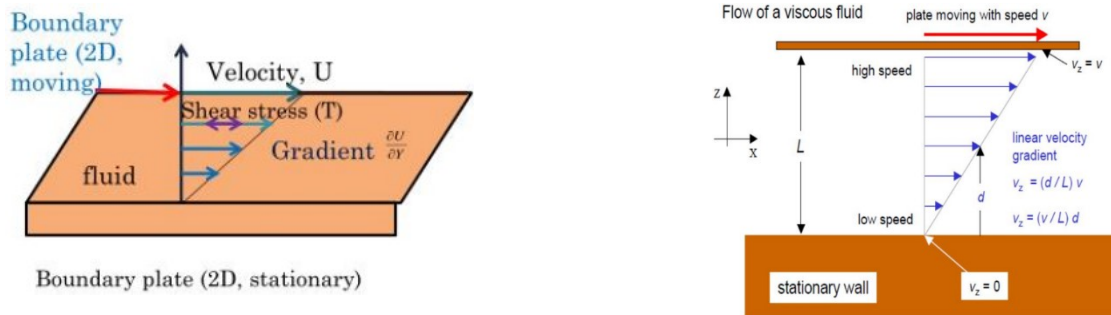


Fig.I.7 Schematics to describe the shear stress in fluid mechanics(Genick Bar–Meir, 2013)

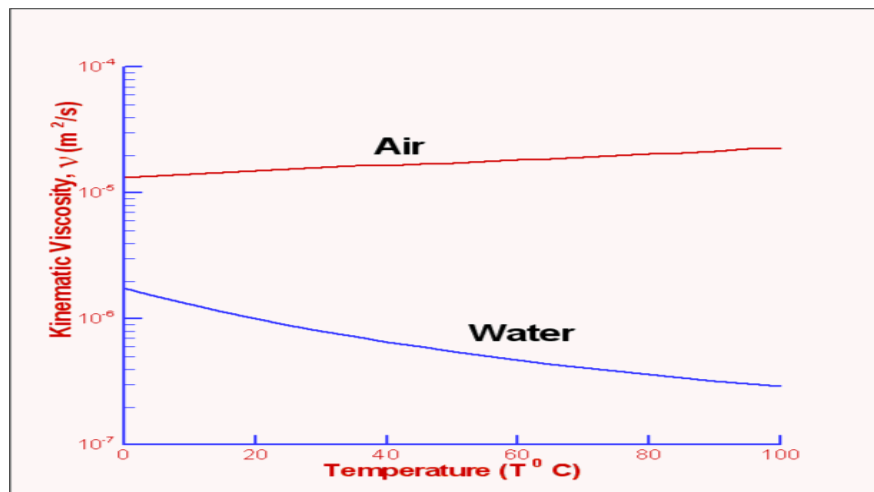


Fig.I.8 Dependence of Viscosity with temperature (Papanastasiou et al., 2021a)

Figure I.8 shows that Viscosity tends to decrease when the temperature increases.

Table I.1: Some dynamic viscosities μ and kinematics ν at 20°C et P = 1atm

FLUID	AIR	WATER	GLYCERIN	GREASE
μ [Pa. s]	18.10^{-6}	1.10^{-3}	0.870	10^3
ν [m^2/s]	15.10^{-6}	1.10^{-6}	635.10^{-6}	1

Observation: Viscous friction always involves energy dissipation

1.5.5. Thermal conductivity k

It is a measure of the ability of a material to conduct heat

$$q = -k \nabla T \quad (1.5)$$

For conduction heat transfer in the x -direction through a surface normal to the x -direction, as a scalar equation:

$$q_x = -k \frac{dT}{dx} \quad (1.6)$$

The minus sign satisfies the convention that heat flux is positive in the direction of decreasing temperature.

Thermal conductivity k [W/m.k] is a measure of its ability to conduct heat, depends on material type, temperature, pressure among other factors.

1.5.6. Mass diffusivity

Mass diffusivity, often denoted as D or D_m , refers to the rate at which particles or molecules of a substance spread out within another substance, typically in a fluid medium. Diffusion is the spread of gases, liquids, or solids from areas of high concentration to areas of low concentration.

The diffusion coefficient for solids tends to be much lower than the diffusion coefficient for liquids and gasses.

Mass diffusivity is influenced by several factors, including temperature, pressure, and the properties of the diffusing substance and the medium. It is commonly determined experimentally or estimated using theoretical models and correlations.

It is an essential parameter for analyzing and predicting the rates of diffusion and transport of species in different systems and understanding how they mix or disperse over **time**.

The most widely used model to describe mass diffusion is Fick's Law of Diffusion (Paul et al., 2014).

$$J = -D(dC / dx) \quad (I.7)$$

Where:

- 💧 J is the diffusion flux (mol/m²·s), representing the rate of diffusion.
- 💧 D is the mass diffusivity (m²/s), which quantifies how easily a substance diffuses.
- 💧 dC/dx is the concentration gradient (mol/m³·m), indicating how the concentration of the diffusing substance changes over distance.

The negative sign indicates that the flux is directed from high to low concentration.

Mass diffusivity plays a critical role in various scientific and industrial applications such as:

Chemical Engineering; Pharmaceutical Industry; Biological Systems; Environmental Science; Material Science(Ogunsona et al., 2018)

I.6. Body Forces vs surface forces

Understanding the various forces acting on fluid particles is essential when studying fluid mechanics. Every fluid particle has the potential to:

- 💧 Surface forces (pressure, friction, viscous) that are generated by contact with other particles or a solid surface;
- 💧 and body forces (such as gravity and electromagnetic) acting on the masses within the control volume.

Surface forces on a fluid particle leads to stresses, which is useful for describing how forces acting on the boundaries of a medium (fluid or solid) are transmitted throughout the medium.

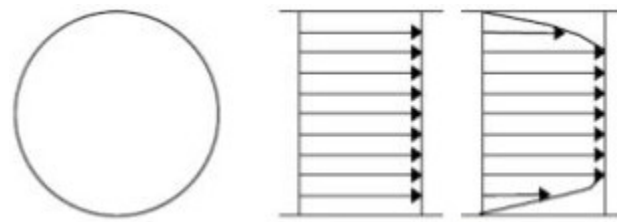
I.7. Ideal fluid vs Real fluid

An ideal fluid is a theoretical concept. We define it as a fluid that is both incompressible and non-viscous.

This means a few key things(Chanson, 2009):

- 💧 **Incompressible:** Its volume doesn't change under pressure. Think of trying to squeeze a balloon filled with water – it's really hard! Ideal fluids behave similarly.
- 💧 **Non-viscous:** It has no internal friction. Imagine pouring honey versus water; honey is more viscous. An ideal fluid would flow without any of this resistance.
- 💧 **Infinite Bulk Modulus:** This means it takes an infinite amount of pressure to compress it.
- 💧 **No Surface Tension:** Surface tension is what allows water droplets to form. Ideal fluids don't have this property.
- 💧 **No Resistance to Shear or Tensile Forces:** Ideal fluids can't resist internal forces that try to deform or pull them apart.

It's important to note: **Ideal fluids are imaginary and don't exist in nature.** They are simply a useful simplification for theoretical calculations. They exhibit smooth, **laminar flow**, which is depicted below:



Pipe	Ideal Flow	Real Flow
	Ideal	Real
Friction	$= 0$	$\neq 0$
viscosity	$\mu = 0$	$\mu \neq 0$
Energy loss	$= 0$	$\neq 0$

Fig.I.9 Velocity distribution of pipe flow

Real fluids possess properties that ideal fluids lack:

- 💧 **Compressible:** Their volume can change under pressure. While liquids are not as easily compressible as gases, they are still compressible to some extent.
- 💧 **Viscous:** They have internal friction, meaning they resist flow. This is why some fluids move more slowly than others.
- 💧 **Surface Tension:** Real fluids exhibit surface tension.
- 💧 **Friction Effects:** Real fluids experience friction as they move and interact with surfaces.
- 💧 **Turbulent Flow:** Real fluids often exhibit turbulent flow, which is chaotic and less predictable, unlike the smooth flow of ideal fluids.

Examples of real fluids are everywhere: kerosene, petrol, and even castor oil.

Chapter II. FLUID STATICS

II.1. Learning Objectives

After completing this chapter, you should be able to:

- 💧 Determine the pressure at various locations in a fluid at rest.
- 💧 Explain the concept of manometers and apply appropriate equations to determine pressures.
- 💧 Calculate the hydrostatic pressure force on a plane or curved submerged surface.
- 💧 Calculate the buoyant force and discuss the stability of floating or submerged objects.

II.2. Introduction

The area of fluid mechanics that deals with fluids at rest and the forces they exert on their surroundings is called **fluid statics**, or **hydrostatics**. Static fluids may seem simple, but their behavior is governed by fundamental principles that have a wide range of applications in the physical sciences and engineering. All shear stresses disappear in a fluid at rest, leaving only normal stresses—related to pressure—acting on fluid elements. Consequently, the primary variable in fluid statics becomes **pressure**. Analyzing a range of systems, from straightforward containers to intricate hydraulic structures, requires an understanding of how pressure changes within a fluid and how it is transmitted.

The definition of **pressure** and its physical interpretation as a **scalar field** acting isotopically in all directions are covered at the beginning of this chapter. Next, the **fundamental law** of fluid statics is established, which links changes in pressure to the forces of the body acting on the fluid, particularly gravity. The **hydrostatic** pressure distribution, which explains how pressure rises with **depth** in a fluid under gravitational **equilibrium**, is derived as a result.

Common tools like **piezometers** and **manometers** are also used to introduce the idea of **pressure measurement**. These devices are crucial for both industrial and laboratory applications and offer useful techniques for measuring pressure differences.

The examination of the **forces** that fluids apply to submerged surfaces takes up a large portion of the chapter. The resulting hydrostatic force and its application point, referred to as the **center of pressure**, can be found by integrating the pressure distribution across a surface. The design and analysis of structures like tanks, gates, dams, and **underwater surfaces** depend on these ideas.

Archimedes' principle, which asserts that a **body submerged** in a fluid experience an **upward force** equal to the weight of the **displaced fluid**, is then used to analyze the **buoyancy principle**. With applications ranging from ship design to offshore structures, this principle serves as the foundation for comprehending the stability and equilibrium of floating and submerged bodies.

Throughout the chapter, emphasis is placed on the physical interpretation of the results and their relevance to engineering practice. Although the governing equations are relatively simple compared to those of fluid dynamics, their correct application requires a clear understanding of pressure distribution and force balance.

By the end of this chapter, the reader will be able to calculate pressure changes, assess hydrostatic forces, analyze fluid systems at rest, and apply buoyancy concepts to real-world issues. These ideas serve as an essential basis for the study of fluid motion and dynamics that follows.

II.3. Fluid statics

Fluid statics is concerned with the balance of forces which stabilize fluids at rest. In the case of a liquid, as the pressure largely changes according to its height, it is necessary to take its depth into account. Furthermore, even in the case of relative rest (e.g., the case where the fluid is stable relative to its vessel even when the vessel is rotating at high speed), the fluid can be regarded as being at rest if the fluid movement is observed in terms of coordinates fixed upon the vessel (Nakayama, 2018)

II.4. Pressure

The term pressure refers to the normal force per unit area at a specific point acting on a specific plane within the fluid mass of interest.

II.4.1. Unit of pressure

The pressure unit pascal is too small for most pressures encountered in practice. Therefore, its multiples kilopascal (1 kPa = 10^3 Pa) and Mega Pascal (1 MPa = 10^6 Pa) are commonly used. Three other pressure units commonly used in practice, especially in Europe, are bar, standard atmosphere, and kilogram-force per square centimeter:

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 101.325 \text{ Pa} = 1.01325 \text{ bars}$$

$$1 \text{ kgf/cm}^2 = 9.807 \text{ N/cm}^2 = 9.807 \times 10^4 \text{ N/m}^2 = 9.807 \times 10^4 \text{ Pa} = 0.9807 \text{ bar} = 0.9679 \text{ atm}$$

Note the pressure units bar, atm, and kgf/cm² are almost equivalent to each other (Cengel & Cimbala, 2017).

II.4.2. Measurement of Pressure

Pressure measured relative to atmospheric pressure is called gauge pressure; it is what a gauge measures if the gauge reads zero before being used to measure the pressure. Absolute pressure is zero in a volume that is void of molecules, an ideal vacuum. (Potter & Wiggert, 2021)

Absolute pressure is related to gauge pressure by the equation:

$$\text{gauge pressure} = \text{absolute pressure} - \text{atmospheric pressure}$$

where atmospheric pressure is the atmospheric pressure at the location where the pressure measurement is made; this atmospheric pressure varies considerably with elevation as shown in figure II.1

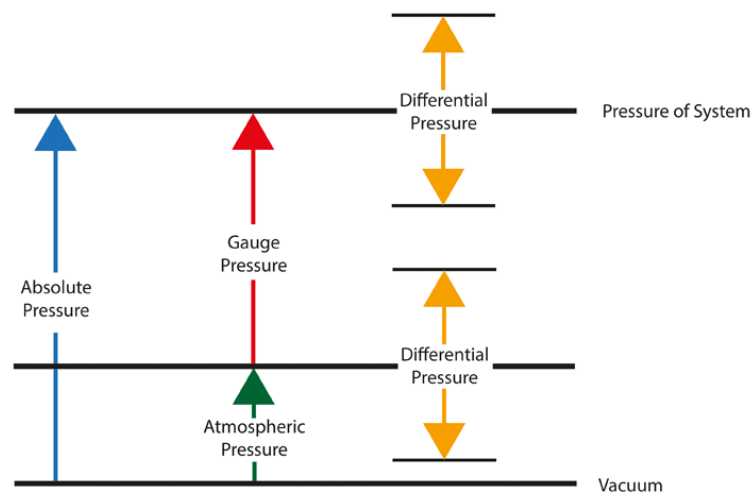


Fig.II.1 Graphical representation of gage and absolute pressure (Higham & Paros, 2003)

Absolute pressures are always positive, but gauge pressures can be either positive or negative depending on whether the pressure is above atmospheric pressure (a positive value) or below atmospheric pressure (a negative value). A negative gauge pressure is also referred to as a suction or vacuum pressure.

For example, 68, 947 kPa (abs) could be expressed as -32.405 kPa (gage), if the local atmospheric pressure is 101.353 kPa, or alternatively 32.405 kPa suction or 32.405 kPa vacuum. (Munson et al., 2013)

II.4.3. Characteristics of pressure

The pressure has the following three characteristics.

1. The pressure of a fluid always acts perpendicular to the wall in contact with the fluid.
2. The values of the pressure acting at any point in a fluid at rest are equal regardless of its direction.

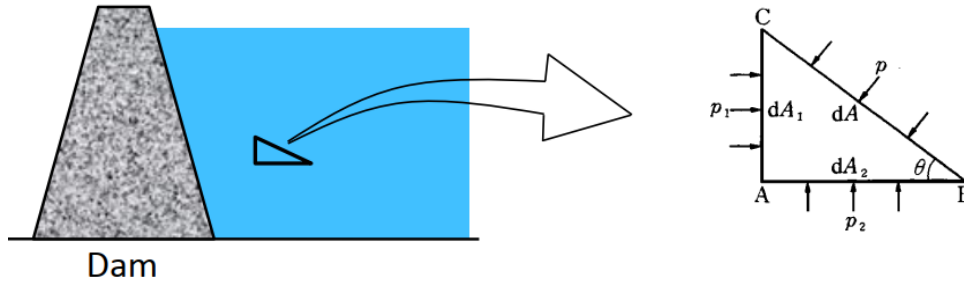


Fig.II.4 Pressure acting on a minute triangular prism (Cüneyt, 2012)

II.4.4. Variation of Pressure with Depth

Pressure in a fluid increase with depth because more fluid rests on deeper layers, and the effect of this “extra weight” on a deeper layer is balanced by an increase in pressure (figure.II.4).

Consider a rectangular fluid element of height Δz , length Δx , and unit depth ($\Delta y = 1$ into the page) in equilibrium. Assuming the density of the fluid ρ to be constant, a force balance in the vertical z -direction gives:

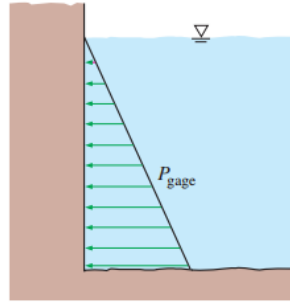


Fig.II.5 Pressure of a fluid at rest(Cengel & Cimbala, 2019)

$$\sum F_z = m \cdot a_z = 0 \quad (\text{II.2})$$

$$P_1 \Delta x \Delta y - P_2 \Delta x \Delta y - \rho g \Delta x \Delta y \Delta z = 0 \quad (\text{II.3})$$

where $\rho g \Delta x \Delta y \Delta z = mg$ is the weight of the fluid element and $\Delta z = z_2 - z_1$

Dividing by $\Delta x \Delta y$ and rearranging gives

$$\Delta P = P_2 - P_1 = -\rho g \Delta z = -(\gamma_s) \Delta z \quad (\text{II.4})$$

Where γ_s is the specific weight of the fluid

A relation for the variation of pressure with elevation can be obtained by dividing Eq. II.4.by Δz , and taking the limit as $\Delta z \rightarrow 0$. These yields

$$\frac{dp}{dz} = -\rho g \quad (\text{II.5})$$

Note that dp is negative when dz is positive since pressure decreases in an upward direction. When the variation of density with elevation is known, the pressure difference between any two points 1 and 2 can be determined by integration to be

$$\Delta P = P_2 - P_1 = - \int_1^2 \rho g dz \quad (\text{II.6})$$

II.5. Fundamental Principal of Hydrostatic

To integrate the above equation to get $p(z)$ we need to know how do ρ and g change with z .

💧 It's safe to consider $g \approx \text{Cste}$. At sea level it is 9.807 m/s^2 and at 14 km altitude it is 9.764 m/s^2 .

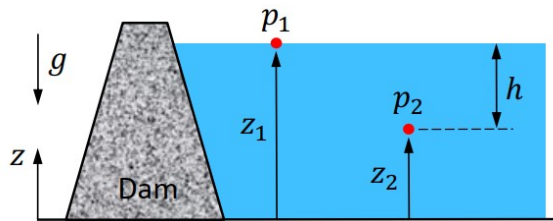


Fig.II.6 Variation of pressure with elevation

- Also, for simplicity let's consider constant density (incompressible fluid).

If ρg is constant $dp/dz = -\rho g$ can be integrated to give:

$$p + \rho g z = \text{constant} \quad (\text{II.7})$$

$$p_1 + \rho g z_1 = p_2 + \rho g z_2 \quad (\text{II.8})$$

$$p_2 = p_1 + \rho g (z_1 - z_2) \quad (\text{II.9})$$

$$p_2 = p_1 + \rho g h \quad (\text{II.10})$$

II.5.1. Pascal's law

As long as we remain in the same continuous fluid and the fluid is at rest, [Pascal's law](#) enables us to "jump" between [fluid columns](#) in [manometers](#) without worrying about pressure changes. The pressure at any location can therefore be calculated by adding or removing $\rho g h$ terms as we move closer to the point of interest, starting from a point of known pressure. For instance, the pressure at the bottom of the tank in Figure.II.7 can be calculated by starting at the free surface, where the pressure is P_{atm} , descending until we reach point 1 at the bottom, and then setting the result to P_1 .

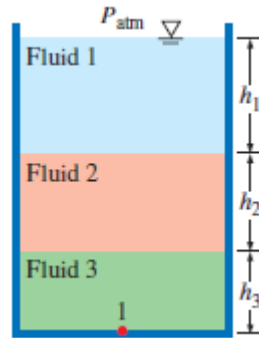


Fig.II.7 An apparatus containing various non-mixed fluids.

It gives:

$$p_{atm} + \rho_1 \cdot g \cdot h_1 + \rho_2 \cdot g \cdot h_2 + \rho_3 \cdot g \cdot h_3 = P_1 \quad (\text{II.11})$$

II.6. Manometers

II.6.1. Barometer

In 1643 [Torricelli](#) demonstrated that atmospheric pressure can be measured using a [mercury barometer](#). Greek word “baros” means weight.

The mercury barometer is an example of one [type of manometer](#), but there are many other configurations possible depending on the particular application.

The pressure at point B in figure II.8 is equal to the atmospheric pressure, and the pressure at point C can be taken to be zero since there is only mercury vapor above point C and the pressure is very low relative to P_{atm} and can be neglected to an excellent approximation.

Writing a force balance in the vertical direction gives:

$$p_{atm} = \rho \cdot g \cdot h \quad (\text{II.12})$$

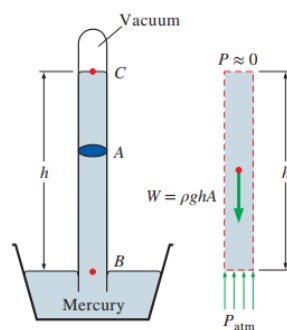


Fig.II.8 The basic barometer pressure

Note that the length and the cross-sectional area of the tube have no effect on the height of the fluid column of a barometer.

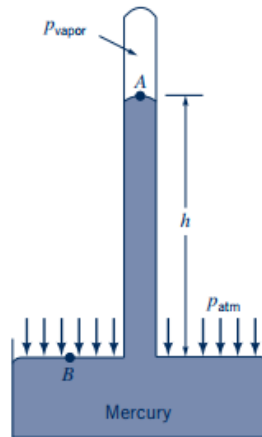


Fig.II.9 Mercury barometer

Atmospheric pressure is usually measured with a mercury barometer, which in its simplest form consists of a glass tube closed at one end with the open end immersed in a container of mercury as shown in figure.II.9.

The tube is initially filled with mercury 1 inverted with its open end up 2 and then turned upside down (open end down), with the open end in the container of mercury. The column of mercury will come to an equilibrium position where its weight plus the force due to the vapor pressure (which develops in the space above the column) balances the force due to the atmospheric pressure.

$$p_{\text{atm}} = \gamma h + p_{\text{vapor}} \quad (\text{II.13})$$

where γ is the specific weight of mercury. For most practical purposes the contribution of the vapor pressure can be neglected since it is very small [for mercury, the fluid most commonly used in barometers, $p_{\text{vapor}} = 344,7380 \text{ kPa (abs)}$ at a temperature of 20°C], so that it is conventional to specify atmospheric pressure in terms of the height, h , in millimeters or inches of mercury. Note that if water were used instead of mercury, the height of the column would have to be approximately 34 ft (10363 mm) rather than 29.9 in (759.46 mm) of mercury for an atmospheric pressure of 14.7 psia (101.353 kPa)!

This is shown to scale in the figure in the margin. The concept of the mercury barometer is an old one, with the invention of this device attributed to Evangelista Torricelli in about 1644.

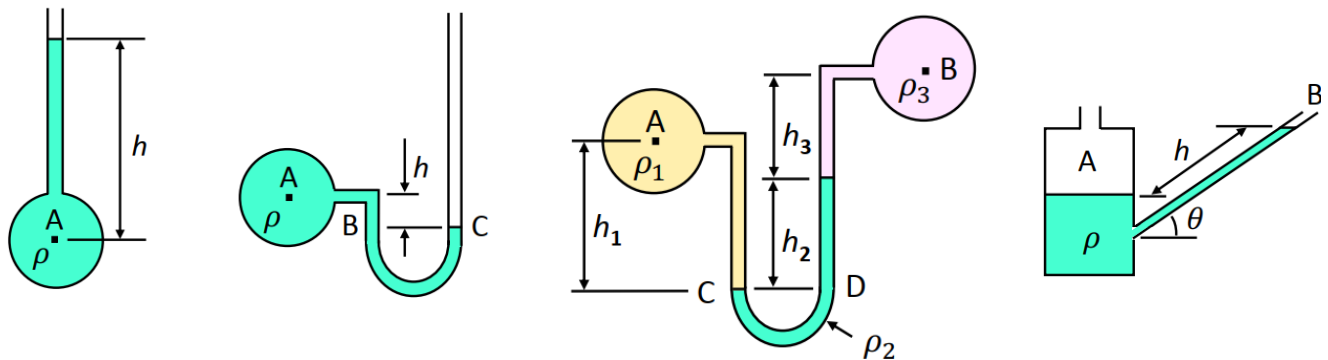
II.6.2. Manometer

A manometer is an instrument that uses a column of liquid to measure pressure, rather than using a pressure gage.

Manometers are used to measure pressure differences using liquid columns in tubes.

Working principles are:

- any two points at the same elevation in a continuous liquid have the same pressure.
- pressure increases as ρgh as one goes down in a liquid column.



$$p_A = p_{atm} + \rho gh \quad p_A + \rho gh = p_C = p_{atm} \quad p_A + \rho_1 gh_1 = p_C = p_D = p_B + \rho_3 gh_3 + \rho_2 gh_2 \quad p_B + \rho gh \sin \theta = p_A$$

Fig.II.10 Measuring Pressure with Manometers (Cüneyt, 2012)

II.7. Buoyancy

II.7.1. Archimedes' Principle

A body wholly or partially immersed in a fluid will experience a buoyant force equal to the weight of the fluid displaced.

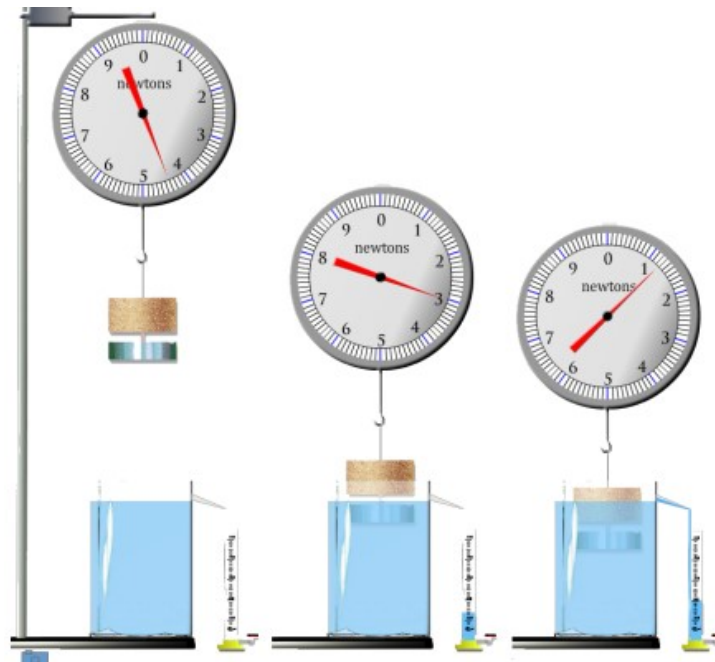


Fig.II.11 The Buoyancy Apparatus (Graf, 2004)

The buoyant force is caused by the increase of pressure with depth in a fluid.

The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume

$$p_g = p_d \text{ (no lateral force)}$$

$$p_b = p_h + \rho g h \text{ (force directed upwards)}$$

$$B = (p_b - p_h)A = \rho g h A = \rho g V \quad (\text{II.14})$$

B = Archimedes' thrust = the fluid's weight that the object is displacing.

$$\vec{F}_{Arch} = \vec{F}_{gravit}$$

$$\vec{F}_{Arch} = \rho_{fluid} \cdot V_{liqu\ imm} \cdot \vec{g} = \rho_{Corps} \cdot V_{immerg} \cdot \vec{g}$$

$$F_B = W \quad (\text{II.15})$$

$$\rho_f g V_{Sub} = \rho_{avg.body} g V_{Total}$$

$$\frac{V_{Sub}}{V_{Total}} = \frac{\rho_{avg.body}}{\rho_f} \quad (\text{II.16})$$

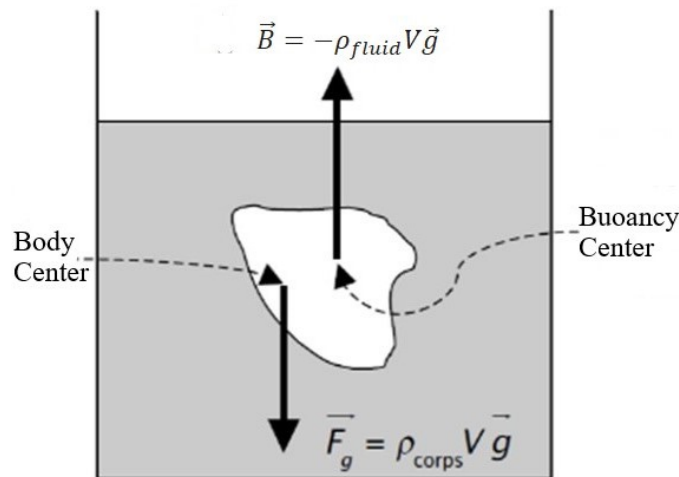


Fig.II.12 Forces acting on immersed body

Therefore, the submerged volume fraction of a floating body is equal to the ratio of the average density of the body to the density of the fluid. Note that when the density ratio is equal to or greater than one, the floating body becomes completely submerged.

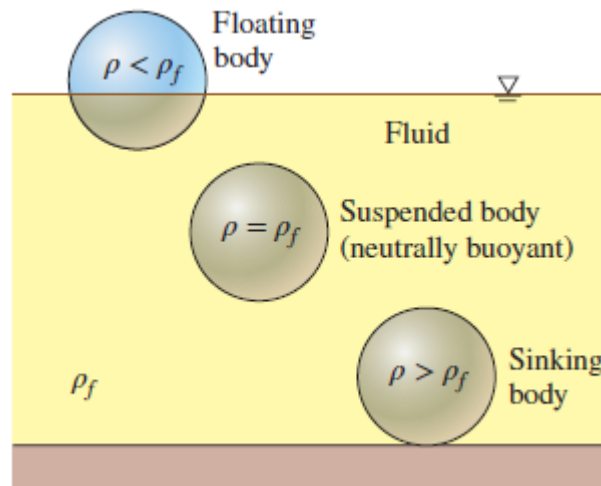


Fig.II.13 Influence of density on immersion

A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its average density relative to the density of the fluid.

II.7.2. Center of gravity and buoyancy

- 💧 **Center of Gravity (COG)** of any object can be defined as the theoretical point at which the object's total weight is assumed to be concentrated due to the presence of gravitational forces.
- 💧 **Center of buoyancy (COB)** can be defined as the theoretical center of the immersed part of an object where the upward buoyant forces act on

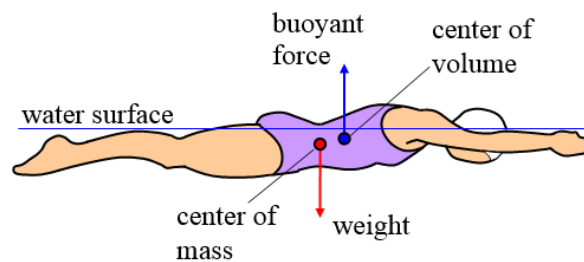


Fig.II.14 Center of gravity and buoyancy

II.7.3. Stability of Immersed Bodies

The stability of an immersed body depends on the relative locations of the center of gravity G of the body and the center of buoyancy B that is the centroid of the displaced volume.

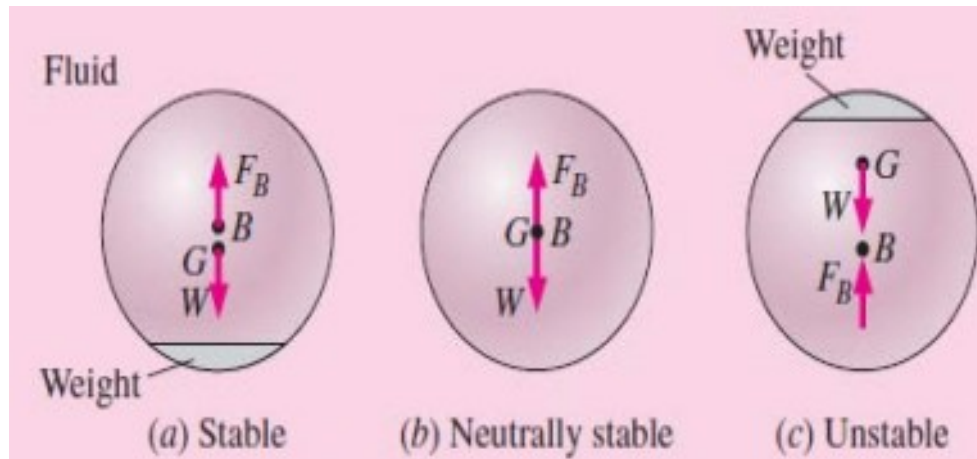


Fig.II.15 Stability of immersed bodies cases

- The body is stable if the body is bottom heavy (G below B); A disturbance produces restoring moment to return the body to its original stable position (E.g., submarines or hot-air balloons)
- Bodies of homogeneous density are neutrally stable, for which G and B coincide.
- The body is unstable if G is directly above B; any disturbance will cause this body to turn upside down.

II.8. Problems Set

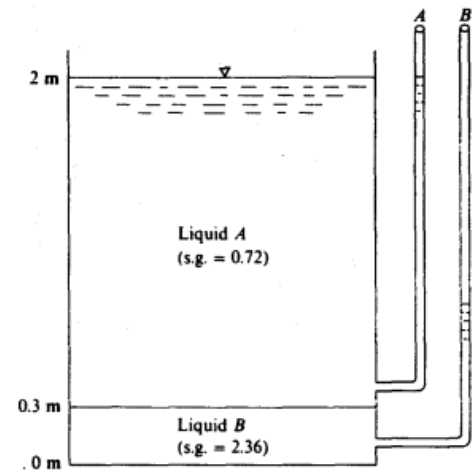
Exercise N°1:

The figure below represents an open tank, equipped with two open piezometric tubes and filled with two immiscible liquids A and B.

Find:

- 1) The elevation Z_A of the liquid surface in piezometer A,
- 2) The elevation Z_B of the liquid surface in piezometer B,
- 3) The total pressure P_C (in bar) at the bottom of the tank.

NB: SG is specific gravity (density)

**Solution:**

- a) Liquid A will simply rise in piezometer A to the same elevation as liquid A in the tank (i.e., to elevation 2 m).
- b) Liquid B will rise in piezometer B to elevation 0.3 m (as a result of the pressure exerted by liquid B) plus an additional amount as a result of the overlying pressure of liquid A.

The overlying pressure can be determined by:

$$p = \gamma h = [(0.72)(9.79)](2 - 0.3) = 11.98 \text{ kN/m}^2.$$

The height liquid B will rise in piezometer B as a result of the overlying pressure of liquid A can be determined by

$$h = p/\gamma = 11.98/[(2.36)(9.79)] = 0.519 \text{ m}.$$

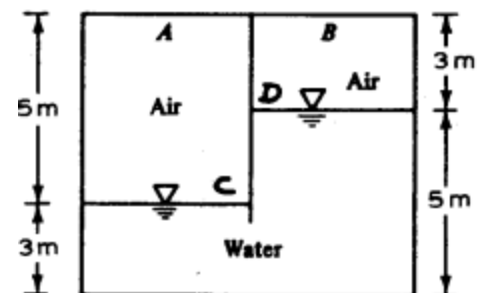
Hence, liquid B will rise in piezometer B to an elevation of 0.3 m + 0.519 m, or 0.819 m.

Exercise N°2 :

The closed tank in figure below is at 20°C. if the pressure at point A is 98 Kpa, what is the absolute pressure at point B? what percent error results from neglecting the specific weight of the air?

$$\rho_{\text{Air}} = 1.2041 \text{ kg/m}^3 \text{ at } 20^\circ\text{C}$$

$$\rho_{\text{water}} = 998.29 \text{ kg/m}^3 \text{ at } 20^\circ\text{C}.$$



Solution :

$$p_A + \gamma_{air} h_{AC} - \gamma_{H_2O} h_{DC} - \gamma_{air} h_{DB} = p_B$$

$$98 + (0.0118)(5) - (9.790)(5 - 3) - (0.0118)(3) = p_B = 78.444 \text{ kPa.}$$

Neglecting air,

$$p_B = 98 - (9.790)(5 - 3) = 78.420 \text{ kPa;}$$

$$\text{error} = (78.444 - 78.420)/78.444 = 0.00031, \text{ or } 0.031\%.$$

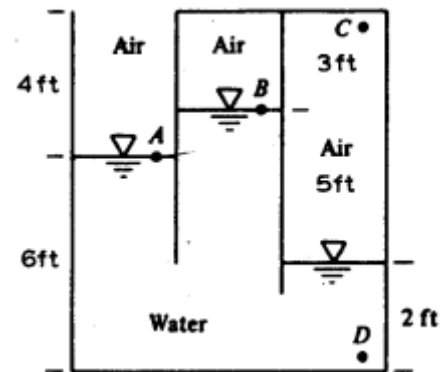
Exercise N°3:

The system in the figure is at 20°C. If the pressure at point A is 138.8527 kPa, determine the pressure at point B, C and D.

$$\rho_{Air} = 1.2041 \text{ kg/m}^3 \text{ at } 20^\circ\text{C}$$

$$\rho_{Water} = 998.29 \text{ kg/m}^3 \text{ at } 20^\circ\text{C}$$

NB: 1 ft = 30.48 cm.



Solution

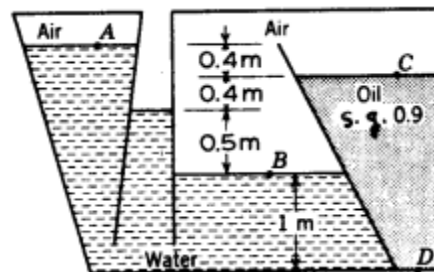
$$p_B = 2900 + (62.4)(4 - 3) = 2838 \text{ lb/ft}^2$$

$$p_D = 2900 + (62.4)(6) = 3274 \text{ lb/ft}^2$$

$$p_C = 2900 + (62.4)(6 - 2) - (0.075)(5 + 3) = 3149 \text{ lb/ft}^2$$

Exercise N°4:

Calculate the pressure, in kPa, at A, B, C, and D.



Solution :

$$p_A = -(0.4 + 0.4)(9.790) = -7.832 \text{ kPa};$$

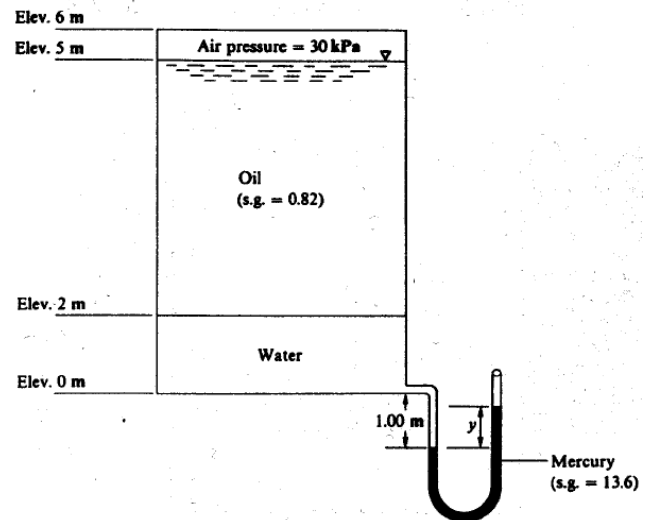
$$p_B = (0.5)(9.790) = 4.895 \text{ kPa};$$

Neglecting air, $p_C = p_B = 4.895 \text{ kPa}$;

$$p_D = 4.895 + (0.9)(9.790)(1 + 0.5 + 0.4) = 21.636 \text{ kPa}$$

Exercise N°5:

A manometer is attached to a tank containing three different fluids, as shown in the figure. What will be the difference in elevation of the mercury column in the manometer?



Solution:

$$30 + [(0.82)(9.79)](5 - 2) + (9.79)(2 - 0) + (9.79)(1.00) - [(13.6)(9.79)]y = 0$$

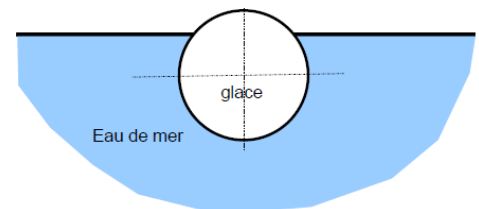
$$\text{So } y = 0.627 \text{ m}$$

Exercise N°6

Ice at -10°C has a density $\rho_{ice} = 995 \text{ kg/m}^3$. A spherical iceberg of 1000 tonnes float on the surface of the water. Sea water has a density $\rho = 1025 \text{ kg/m}^3$

Required work:

- 1) Determine the fraction F of the submerged volume?
- 2) What will F be if the ice had a cubic shape?



Solution:**Given**

- 💧 Mass $m = 1000 \text{ tonnes} = 1 \times 10^6 \text{ kg}$
- 💧 Density of ice $\rho_{\text{ice}} = 995 \text{ kg/m}^3$
- 💧 Density of sea water $\rho_{\text{sw}} = 1025 \text{ kg/m}^3$

1) Fraction F of the submerged volume

For a floating body at equilibrium the fraction submerged is

$$F = \frac{V_{\text{displaced}}}{V_{\text{total}}} = \frac{\rho_{\text{ice}}}{\rho_{\text{sw}}}$$

$$\text{So } F = \frac{995}{1025} = 0.9707317 \dots \approx 0.9707.$$

As a percentage: $F \approx 97.07\%$ of the iceberg's volume is submerged.

What if the ice had a cubic shape?

The submerged fraction for a floating object depends only on densities (not on shape), so the fraction F is the same:

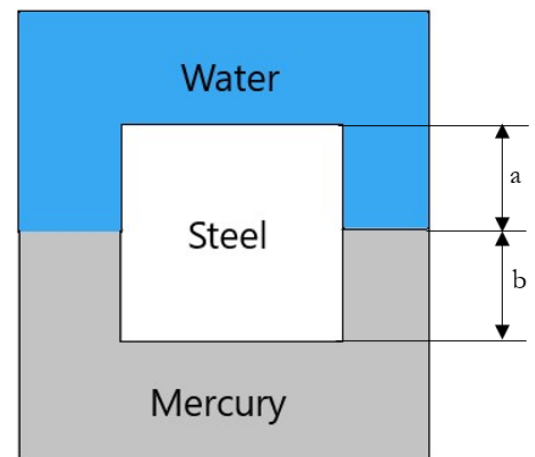
$$F_{\text{cube}} = \frac{\rho_{\text{ice}}}{\rho_{\text{sw}}} = 0.9707 \approx 97.07\%$$

Exercise N°7

A block of parallelepiped steel floats at a water-mercury interface as shown below.

We note d_A and d_M the respective densities of steel and mercury

- 1) Calculate the ratio of distances b/a .
- 2) Digital application : $d_A = 7,85$ and $d_M = 13$.



Solution**Given**

A steel block (density d_A) floats **at the interface** between **water** (density d_W) and **mercury** (density d_M).

- Part immersed in **water**: height a
- Part immersed in **mercury**: height b
- Total immersed height = $a + b$

Principle

For equilibrium, the **weight of the block** equals the **sum of buoyant forces** from water and mercury:

$$\text{Weight of block} = \text{Buoyant force (water)} + \text{Buoyant force (mercury)}$$

Express mathematically

If the block has cross-sectional area S :

$$d_A g \cdot S(a + b) = d_W g \cdot S \cdot a + d_M g \cdot S \cdot b$$

$$d_A(a + b) = d_W a + d_M b$$

Simplify to find $\frac{b}{a}$:

$$\begin{aligned} d_A a + d_A b &= d_W a + d_M b \\ (d_A - d_W)a &= (d_M - d_A)b \end{aligned}$$

$$\boxed{\frac{b}{a} = \frac{d_A - d_W}{d_M - d_A}}$$

Numerical application

$$\text{💧 } d_W = 1$$

$$\text{💧 } d_A = 7.85$$

$$\text{💧 } d_M = 13$$

$$\text{Then: } \frac{b}{a} = \frac{7.85-1}{13-7.85} = \frac{6.85}{5.15} = 1.33$$

$$\text{Final Results: } \boxed{\frac{b}{a} = 1.33}$$

That means the part of the block submerged in mercury is about **1.33 times** higher than the part submerged in water.

Chapter III. FLUID KINEMATICS

III.1. Prerequisite Knowledge

Students should already know:

Mathematics

- Basic derivatives: $\frac{df}{dx}$, partial derivatives
- Notions of gradient and spatial variation of scalar and vector fields
- Functions of several variables and partial derivatives.
- Simple integrals

Physics

- Notions of **gradient** and spatial variation of scalar and vector fields
- Basic kinematics (velocity, acceleration)
- Pressure concept (no fluid mechanics background needed)

III.2. Introduction

Fluid kinematics aims to **describe** the evolution of a **fluid medium** in space-time independently of the causes and laws that govern it. Therefore, without considering the forces at their source, we will concentrate on the motion of fluids with respect to time in this chapter.

We first begin by defining a "fluid **particle**" (or, more broadly, a "**continuous medium** particle" for any continuous medium). The motion of this **small volume** of "labeled" matter is what we "follow." To this fluid particle, we associate kinematic quantities (position, velocity, acceleration) and thermodynamic quantities (density, temperature, pressure, etc.).

The **study** of fluid motion without direct reference to forces—plays a crucial role in the **efficient design**, operation, and safety of piping systems, ventilation, pumps and turbines, and heat exchangers.

Example: In industrial piping for chemical plants, knowing the **velocity profile** prevents erosion of the pipe walls and ensures safe transport of reactive fluids, by determining the critical flow velocity and understand mechanisms of the phenomenon so that it can be used for proper applications(Fannelöp, 2013; Wang & Zheng, 2021) .

III.3. Description of Fluid Motion

III.3.1. Velocity field

When we describe the flow of a fluid like water, we may think of the movement of individual particles. These particles interact with each other through forces. We could then apply our laws of motion to each individual particle in the fluid but because the number of particles is very large, this would be an extremely difficult computation problem.

Instead, we shall begin by mathematically describing the state of moving fluid by specifying the velocity of the fluid at each point in space and at each instant in time.

In mathematics and physics, a **scalar** is a quantity that only has magnitude (size), while a **vector** has both magnitude and direction.

💧 **Scalar** quantities include pure numbers, pressure, temperature and energy.

💧 **Vector** quantities include **velocity**, acceleration, momentum, and forces.

So, we define the **velocity field** as a **vector** field variable.

Pressure field	$P = P(x, y, z, t)$
Velocity field	$\vec{V} = \vec{V}(x, y, z, t)$

The velocity field can be expressed in Cartesian coordinates as follows:

$$\vec{V}(u, v, w) = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k} \quad (\text{III.1})$$

III.4. Lagrangian and Eulerian Description

There are two general approaches in analyzing fluid mechanics problems (Dourmashkin, 2014; Munson et al., 2013). The first method, called the Eulerian method. In this case, the fluid motion is given by completely prescribing the necessary properties (pressure, density, velocity, etc.) as functions of space and time.

From this method we obtain information about the flow in terms of what happens at fixed points in space as the fluid flows through those points.

The second method, called the Lagrangian method, involves following individual fluid particles as they move about and determining how the fluid properties associated with these particles change as a function of time.

III.4.1. Lagrangian approach

The Lagrangian approach involves keeping track of the position vector $\vec{x}_0, \vec{x}_1, \dots$ of each object as shown in figure V.1 and the velocity vector of each object $\vec{V}_0, \vec{V}_1, \dots$, as functions of time (BERIACHE, 2019).

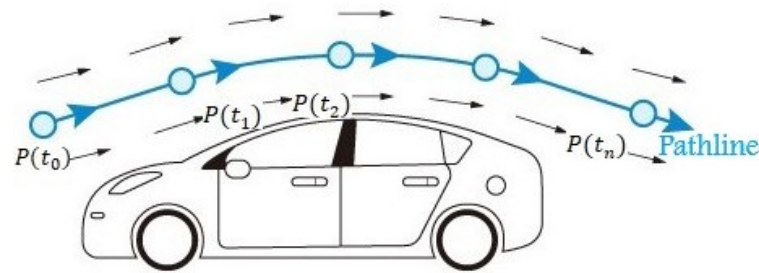


Fig.III.1 Movement of a fluid particle over time

III.4.2. Concept of path line

This leads to the definition of the **"path line"** of a fluid particle "P": it is the set of positions successively occupied by the same particle. It is therefore a video of a chosen particle in the flow.

III.4.3. Eulerian approach

Unlike the Lagrangian description, where particles are identified, the Eulerian description consists of fixing an observation point x in space and recording over time the velocity of the fluid particles passing by this point.

Euler's point of view is more convenient for the experimenter, because one places oneself at a point $M(x, y, z)$ of the fluid and studies the variations of physical quantities (for example, velocity) at different times.

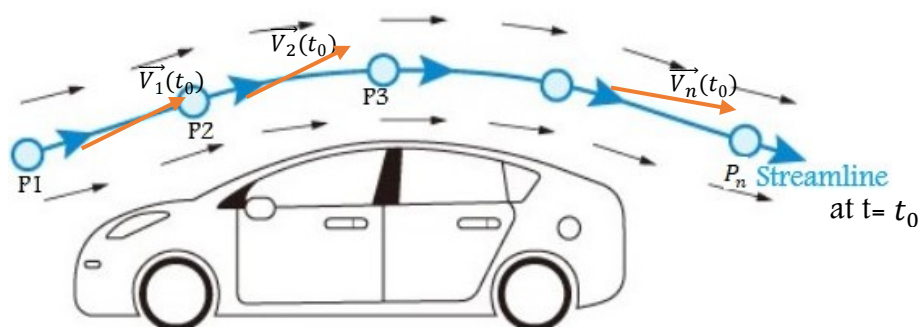


Fig.III.2 Streamline concept

III.4.4. Concept of streamline

A **streamline** is a line which smoothly connects velocity vectors at an instance in time. In other words, an image of the flow characterized by streamlines is like a snapshot of the flow at one moment in time.

III.5. Advantages of Eulerian approach

In fluid mechanics it is usually easier to use the Eulerian method to describe a flow in either experimental or analytical investigations (Bellakhal, 2015).

- 💧 **Lagrangian** information concerns the nature and behavior of fluid parcels.
- 💧 **Eulerian** information concerns fields, i.e., properties like velocity, pressure and temperature that vary in time and space.

For example, suppose an apple falls from a tree. Newton taught us to describe the height and velocity of the apple as functions of time. This is a Lagrangian description.

We could define this event as the "applicability" as (1) at the points in space and time occupied by the apple and (0) everywhere else, then try to formulate a differential equation for it in order to describe it in terms of Eulerian fields.

III.6. Path line and Streamline equation

III.6.1. Path line equation

The differential parametric equations of the trajectories are given by:

$$\begin{cases} \frac{dx}{dt} = u(x, y, z, t) \\ \frac{dy}{dt} = v(x, y, z, t) \\ \frac{dz}{dt} = w(x, y, z, t) \end{cases} \quad (\text{III.2})$$

In these equations, time has become a variable

Example: At a point M (x, y), the fluid velocity is: $\vec{V} = (kx, ky)$

Determine the path line equations of the particle.

Response:

Path line equation

$$\begin{cases} \frac{dx}{dt} = u = kx \\ \frac{dy}{dt} = v = ky \end{cases} \quad \begin{cases} \frac{dx}{x} = k dt \\ \frac{dy}{y} = k dt \end{cases}$$

So $\frac{dx}{x} = \frac{dy}{y}$; $\int \frac{dx}{x} = \int \frac{dy}{y}$ d'où $\ln y = \ln x + \ln C$; $y = e^{\ln x} \cdot e^{\ln C} = A \cdot x$

III.6.2. Stream line Equation

Streamline Equation: With respect to an orthonormal coordinate system, the differential equation of any streamline is written:

$$\frac{dx}{u(x, y, z, t)} = \frac{dy}{v(x, y, z, t)} = \frac{dz}{w(x, y, z, t)} \quad (\text{III.3})$$

with t fixed.

$U(x, y, z, t)$, $V(x, y, z, t)$ et $w(x, y, z, t)$ are the velocity components in the considered coordinate system.

Example: Determine the equations of the streamline network: $\vec{V} \begin{cases} u = 2x - 3z \\ v = 0 \\ w = 3x - 2z \end{cases}$

We have $v=0$; So, the flow lies in the Oxz plane.

Therefore: $\frac{dx}{2x-3z} = \frac{dz}{3x-2z}$

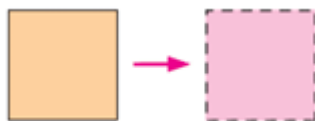
Which gives $(3x - 2z).dx = (2x - 3z).dz$; then $3x.d x - 2d.(xz) + 3zdz = 0$

Finally: $\frac{3}{2}x^2 + \frac{3}{2}z^2 - 2xz = \text{Cste}$

III.7. Deformation of Fluid Elements

A crucial classification of fluid flow involves determining whether the flow is in translation, rotational or irrotational. This distinction plays an important role in many applications, such as fluid dynamics, aerodynamics, and hydrodynamics.

III.7.1. Translation

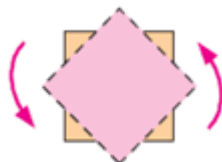


Fluid element moves

No Deformation

Just movement

III.7.2. Rotation (vorticity)

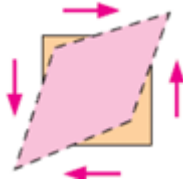


Fluid element rotate

No Deformation

Just movement

III.7.3. Deformation (strain rates)



Shear Strain

Deformation

Shear strain rate can be mathematically defined as in Cartesian coordinates the partial derivative of the velocity components along the principal axes:

$$\dot{\gamma} = \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} \quad (\text{III.4})$$

Where:

u = velocity component along the x directions [m/s]

v = velocity component along the y directions [m/s]

The viscosity of a fluid plays a significant role in determining the shear strain rate.

III.7.4. Shear Stress vs Shear Strain Rate

- 💧 Shear stress in fluids refers to the force applied parallel to the surface of the fluid, divided by the area over which it is applied. It represents the intensity of the internal forces within the fluid that resist the sliding of adjacent layers. Shear stress causes continuous shear deformation in a fluid.
- 💧 Shear strain rate, on the other hand, is a measure of the rate at which adjacent fluid layers deform or move relative to each other under the influence of shear stress. It is the change in shear strain (the deformation) per unit of time.

For Newtonian fluids, the relationship between shear stress and shear strain rate comes from the Newton's law of viscosity. This linear relationship can be expressed as:

$$\tau = \mu \dot{\gamma} \quad (\text{III.5})$$

Where:

- τ = shear stress [Pa]
- μ = dynamic viscosity of the fluid [Pa.s]

Example of flow deformation:

Consider water flowing through a pipe that gradually narrows (a converging section). At the walls, the fluid velocity is zero due to the no-slip boundary condition, while the central core of the flow moves faster. This velocity gradient generates a **shear deformation** in the fluid.

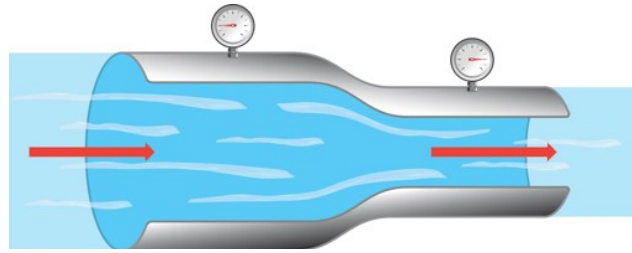


Fig.III.3 Flow of Water Through a Converging Pipe (Beauvais, 1994)

Observations:

1. **Shear Strain Near Walls:** Fluid layers close to the walls deform more because of the high velocity differences between neighboring layers.
2. **Deformation Tensor:** The fluid element experiences a combination of linear strain and shear strain. Rotation of the element (vorticity) occurs but is separated from pure deformation for analysis.
3. **Material Motion:** Following a small fluid element (Lagrangian perspective), the shape continuously changes, elongating along the flow and compressing perpendicular to it while conserving volume for incompressible flows

III.8. Vorticity and Irrotationality

The concept of vorticity and irrotationality are very useful in analyzing many fluid problems.

Vorticity is a type of fluid motion characterized by the presence of **vortices** or **rotational** components within the flow field. Unlike **irrotational** flow, where fluid particles move without rotating, rotational flow involves the swirling or **circular motion** of fluid particles around a central axis (White, 2008).

It is commonly observed in natural phenomena like tornadoes, whirlpools, and turbulent flows. The presence of vortices in rotational flow can have significant effects on fluid behavior, such as changes in pressure distribution, mixing and diffusion rates, and the development of shear forces.

A flow is said to be **rotational** if the fluid particles while moving in the flow direction **rotate about their mass center**.

In fact, the **irrotational** flow exists only in the case of **ideal fluid** for which **no tangential or shear stress** occur. The flow of **low viscosity** fluid may be considered as irrotational flow.

Rotational flow (vorticity vector) is defined as a flow for which we have:

$$\text{curl}(\vec{V}) = \overrightarrow{\text{rot}} \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \neq 0 \quad (\text{III.6})$$

From this equation we can deduce that:

$$\text{curl}(\vec{V}) = \vec{\nabla} \times \vec{V} \quad (\text{III.7})$$

Which implies that, for an incompressible fluid and irrotational flow, the equation will be:

$$\text{div}(\vec{V}) + \frac{\partial \rho}{\partial t} = 0 \quad (\text{III.8})$$

III.9. Flows Properties

III.9.1. Planar flows

In space referred to the direct orthonormal coordinate system (O; x, y, z), the flow considered is said to be planar flow parallel to the plane (O; x, y) if the velocity vector $\vec{V}(u(x, y, z, t), v(x, y, z, t), \omega(x, y, z, t))$ expressed in Euler variables is such that:

$$\begin{cases} u = u(x, y, t) \\ v = v(x, y, t) \\ \omega = 0 \end{cases} \quad (\text{III.9})$$

III.9.2. Steady Flow

A flow is said to be steady (or permanent) if the velocity, expressed in Euler variables, does not explicitly depend on time t. In other words: $\vec{V}(u, v, \omega)$:

$$\begin{cases} u = u(x, y, z) \\ v = v(x, y, z) \\ \omega = \omega(x, y, z) \end{cases} \quad (\text{III.10})$$

Naturally, a flow that is not steady is said to be unsteady. For a steady flow, the streamlines remain fixed over time.

III.9.3. Conservative flow (without fluid production)

The equation for the overall form of the mass conservation principle:

Local equation and steady state:

$$\text{div}(\rho \vec{j}) + \frac{\partial \rho}{\partial t} = 0 \quad (\text{III.11})$$

Steady state: $\partial \rho / \partial t = 0$ hence $\text{div}(\rho \vec{j}) = 0$

III.9.4. incompressible fluid

In case of an incompressible fluid ($\rho = \text{constant}$):

$$\partial \rho / \partial t = 0 \text{ then } \text{div } \vec{V} = 0 \quad (\text{III.12})$$

This equation represents the conservation of volume flow rate for an incompressible fluid.

III.9.5. Acceleration Field

The material acceleration is defined as the acceleration following a fluid particle (Yunus, 2010). Since acceleration is the time derivative of velocity $\vec{V}_{particle}(t) \equiv \vec{V}(x_{particle}(t), y_{particle}(t), z_{particle}(t))$.

The material acceleration can be derived from the definition of material derivative as follows:

$$\begin{aligned} \vec{\Gamma}_{particle}(t) &= \frac{d\vec{V}}{dt} = \frac{d\vec{V}(x_{particle}(t), y_{particle}(t), z_{particle}(t))}{dt} \\ &= \frac{\partial \vec{V}}{\partial t} \frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x_{particle}} \frac{dx_{particle}}{dt} + \frac{\partial \vec{V}}{\partial y_{particle}} \frac{dy_{particle}}{dt} + \frac{\partial \vec{V}}{\partial z_{particle}} \frac{dz_{particle}}{dt} \quad (\text{III.13}) \\ \vec{\Gamma}_{particle}(x, y, z, t) &= \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z} \end{aligned}$$

The tensor model of the acceleration of a fluid particle:

model of the acceleration of a fluid particle:

$$\vec{\Gamma}_{particle}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \quad (\text{III.14})$$

With local acceleration $\frac{\partial \vec{V}}{\partial t}$ and $(\vec{V} \cdot \nabla) \vec{V}$ advective (convective) acceleration

The components of acceleration in Cartesian coordinates:

$$\begin{aligned} \Gamma_x &= \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \\ \Gamma_y &= \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \\ \Gamma_z &= \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \end{aligned} \quad (\text{III.15})$$

III.10. Flow Visualization & Measurement

Flow visualization is an essential technique in engineering that allows engineers and researchers to study and analyze the patterns of fluid flow (Smits, 2012). Whether in aerodynamics, hydrodynamics, or within systems like pipelines and engines, understanding how fluids move is crucial for design, safety, efficiency, and troubleshooting.

III.10.1. Types of Flow Visualization Techniques

Flow visualization can be broadly classified into qualitative and quantitative techniques. Qualitative methods provide visual insight into the flow, generally highlighting structures and behaviors, while quantitative methods involve measurements of flow characteristics like velocity, pressure, and density.

💧 Smoke and Dye Visualization:

This is one of the simplest and most intuitive methods of flow visualization. Colored dyes or smoke are introduced into a flow field, and their dispersion provides visual insight into flow patterns, wakes, vortices, and mixing layers. It is typically used in wind tunnels and water channels.



Fig.III.4 AI generated of Smoke Visualization

💧 Particle Image Velocimetry (PIV):

PIV is a sophisticated technique that uses particles seeded in the flow which are illuminated by a laser sheet. Cameras capture the motion of these particles over a short time interval, and the particle displacement fields are computed to map velocity vectors across the plane of the laser sheet.

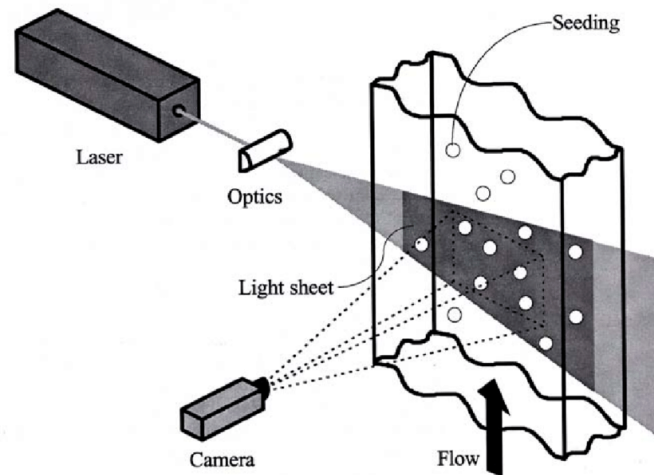


Fig.III.5 Particle Image Velocimetry (Al Makky, 2007)

💧 Laser Doppler Velocimetry (LDV)

LDV utilizes the Doppler shift of laser light scattered by particles flowing with the fluid to measure the velocity at specific points in the flow field. This method is highly accurate and non-intrusive, making it suitable for detailed research studies.

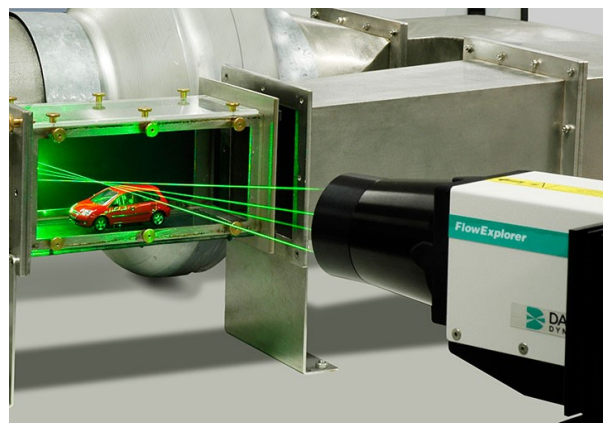


Fig.III.6 Laser Doppler Velocimetry(Beauvais, 1994)

III.10.2. Application Areas

Each flow visualization technique has its unique application areas and is chosen based on the desired information, the phase of the fluid (liquid or gas), and the physical parameters involved.

- 💧 **Aerodynamics:** Techniques like Schlieren photography and smoke visualization are widely used in the study of airflows around objects, helping in the design of aircraft, automobiles, and buildings against wind loads.
- 💧 **Marine Engineering:** Dye and smoke methods help visualize the flow around ship hulls and propellers, crucial for optimizing design for stability and efficiency.
- 💧 **Medical Engineering:** Laser Doppler Velocimetry is used in biomedical research to measure blood flow in cardiovascular research, contributing to advancements in medical diagnostics and treatments.
- 💧 **Environmental Engineering:** Particle Image Velocimetry can be utilized to study sediment transport in rivers and the dispersion of pollutants in air or water, aiding environmental protection efforts.

III.11. Problems Set

Exercise N°1

Consider the flow defined by the following velocity field:

$$\begin{cases} u = x \\ v = -y \end{cases}$$

- State if the flow is curl
- Determine the streamline equation
- Determine the path line equation

Solution:

Consider the velocity field: $\begin{cases} u = x \\ v = -y \end{cases}$

- Curling flow: It is sufficient to check that: $\overrightarrow{\text{curl}} \vec{V} = 0$

Calculation of the curl:

$$\overrightarrow{\text{curl}} \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & 0 \end{vmatrix} = \vec{i} \left(\frac{\partial 0}{\partial y} - \frac{\partial (-y)}{\partial z} \right) - \vec{j} \left(\frac{\partial 0}{\partial x} - \frac{\partial x}{\partial z} \right) + \vec{k} \left(\frac{\partial (-y)}{\partial x} - \frac{\partial x}{\partial y} \right) = 0$$

Therefore, the flow is **irrotational**. The velocity field derives from a potential, and this type of flow is called a **potential flow**.

Thus, the velocity derives from a potential function ϕ , such that: $\vec{V} = -\nabla\phi$

$$\text{So: } d\phi = \frac{\partial\phi}{\partial x} dx + \frac{\partial\phi}{\partial y} dy$$

$$\text{Since } u = \frac{\partial\phi}{\partial x} = x, \text{ we have: } \phi = \frac{x^2}{2} + f_1(y)$$

$$\text{Since } v = \frac{\partial\phi}{\partial y} = -y, \text{ we obtain: } \phi = -\frac{y^2}{2} + f_2(x)$$

$$\text{By identification: } f_1(y) = -\frac{y^2}{2}, f_2(x) = \frac{x^2}{2}$$

$$\text{Thus: } \boxed{\phi(x, y) = \frac{x^2}{2} - \frac{y^2}{2}}$$

1. Streamline equation

For a steady two-dimensional incompressible flow, the equation of the streamlines is: $\frac{dx}{u} = \frac{dy}{v}$

Substituting $u = x$ and $v = -y$: $\frac{dx}{x} = -\frac{dy}{y}$

After integration: $\ln x + \ln y = C \Rightarrow xy = A$ (arbitrary constant)

or: $y = \frac{C}{x}$

Thus, the streamlines are **hyperbolas**.

2. Path line equation

Since: $u = \frac{dx}{dt} = x, v = \frac{dy}{dt} = -y$

We have: $\frac{dx}{x} = dt, \frac{dy}{y} = -dt$

This gives: $\ln x = t + C_1, \ln y = -t + C_2$

Therefore: $t = \ln x - C_1$

From which: $\ln x + \ln y = B$

So: $xy = B$ or $y = \frac{B}{x}$; This is also the equation of a **hyperbola**.

Thus, **in a steady flow, streamlines coincide with the path line**.

Exercise N°2

Consider a velocity field; $\vec{V} = (4 + xy + 2t)\vec{i} + 6x^3\vec{j} + (3xt^2 + z)\vec{k}$;

1. State, justifying your answer with calculations, whether the flow is:

- Planar flow?
- Steady or unsteady flow?
- Compressible,
- Curling

2. Find the acceleration of a fluid particle at (2, 4, -4) and time $t = 3$.

Solution:

The velocity field: $\vec{V} = (4 + xy + 2t)\vec{i} + 6x^3\vec{j} + (3xt^2 + z)\vec{k}$;

1. Compressible means $\text{div } \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \neq 0$

$$\text{div } \vec{V} = \frac{\partial(4+xy+2t)}{\partial x} + \frac{\partial(6x^3)}{\partial y} + \frac{\partial(3xt^2+z)}{\partial z} = (yx^2)\vec{i} + z^2\vec{k}$$

$$\vec{\Gamma}_{\text{particle}}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V}$$

2. Fluid particle acceleration, Tensor notation:

Hence $\vec{\Gamma}_{\text{particle}}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z}$ Either $\frac{\partial \vec{V}}{\partial t} = 2\vec{i} + 6xt\vec{k}$

$$(\vec{V} \cdot \nabla) \vec{V} = u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z} = \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \right) [(4 + xy + 2t)\vec{i} + 6x^3\vec{j} + (3xt^2 + z)\vec{k}]$$

$$= u(y\vec{i} + 18x^2\vec{j} + 3t^2\vec{k}) + v(x\vec{i}) + w(\vec{k})$$

$$= (4y + xy^2 + 2ty + 6x^4)\vec{i} + (72x^2 + 18x^3y + 36tx^2)\vec{j} + (12t^2 + 3xyt^2 + 6t^3 + 3xt^2 + z)\vec{k}$$

Finally, the acceleration field can be expressed by:

$$\vec{\Gamma}_{\text{particle}}(x, y, z, t) = (2 + 4y + xy^2 + 2ty + 6x^4)\vec{i} + (72x^2 + 18x^3y + 36tx^2)\vec{j} + (6xt + 12t^2 + 3xyt^2 + 6t^3 + 3xt^2 + z)\vec{k}$$

The acceleration vector at point (2, 4, -4) and at time t=3s can be determined by substituting the values of x, y, z and t into the equation.

We obtain: $\vec{\Gamma}_{\text{particle}}(2, 4, -4, 3) = 170\vec{i} + 1296\vec{j} + 572\vec{k}$

Exercise N°3

Given the velocity field: $\vec{V} = (5x)\vec{i} + (15y + 11)\vec{j} + (19t^2)\vec{k}$, determine the path line and streamline equation of a particle which is at (4, 6, 2) m, at time t=3s.

Given the velocity field: $\vec{V} = (5x)\vec{i} + (15y+11)\vec{j} + (19t^2)\vec{k}$, at (4,6,2) m, at time $t=3s$.

$$1. \text{ Path line (trajectory of the particle): } \begin{cases} \frac{dx}{dt} = 5x \\ \frac{dy}{dt} = 15y + 11; \\ \frac{dz}{dt} = 19t^2, \end{cases}$$

with initial conditions $x(t=3) = 4$, $y(t=3) = 6$, $z(t=3) = 2$.

- **Solve for $x(t)$:** $\frac{dx}{dt} = 5x \Rightarrow x(t) = C_x e^{5t}$, Use $x(3) = 4$: $C_x = 4e^{-15}$. So

$$\boxed{x(t) = 4 e^{5(t-3)} = 4e^{5t-15} .}$$

- **Solve for $y(t)$:** $\frac{dy}{dt} - 15y = 11$. Integrating factor e^{-15t} : $\frac{d}{dt}(ye^{-15t})1e^{-15t} \Rightarrow ye^{-15t} = -\frac{11}{15}e^{-15t} + C_y$.

Thus $y(t) = -\frac{11}{15} + C_y e^{15t}$. Use $y(3) = 6 = -\frac{11}{15} + C_y e^{45} \Rightarrow C_y = \frac{101}{15}e^{-45}$;

- So $\boxed{y(t) = -\frac{11}{15} + \frac{101}{15} e^{15(t-3)}} .$ (Equivalently $y(t) = -\frac{11}{15} + (6 + \frac{11}{15})e^{15(t-3)}$.);

Solve for $z(t)$: $\boxed{z(t) = \frac{19}{3}t^3 - 169 .}$

So, the particle path line at points (4, 6, 2, $t=3s$) is:

$$\begin{array}{lcl} x(t) & = & 4 e^{5(t-3)}, \\ y(t) & = & -\frac{11}{15} + \frac{101}{15} e^{15(t-3)}, \\ z(t) & = & \frac{19}{3}t^3 - 169. \end{array}$$

2. Streamline at time $t_0 = 3$

A streamline at fixed time t_0 solves: $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$,

with $u = 5x$, $v = 15y + 11$, $w = 19t_0^2$.

Here $t_0 = 3s$, $w = 19 \cdot 3^2 = 171$. Thus: $\frac{dx}{5x} = \frac{dy}{15y+11} = \frac{dz}{171}$.

Relation between x and y : $\frac{1}{5} \frac{dx}{x} = \frac{1}{15} \frac{dy}{y+11/15}$

Integrate: $3 \ln(x) = \ln(y + \frac{11}{15}) + C \Rightarrow y + \frac{11}{15} = C_s x^3$.

The streamline through (4, 6, 2) fixes C_s : $6 + \frac{11}{15} = \frac{101}{15} = C_s \cdot 4^3 = 64C_s \Rightarrow C_s = \frac{101}{960}$.

So the x - y equation of the streamline is: $\boxed{y = \frac{101}{960} x^3 - \frac{11}{15} .}$

Relation between x and z : $\frac{1}{5} \frac{dx}{x} = \frac{dz}{171} \Rightarrow z = \frac{171}{5} \ln x + C'$.

Use $(x, z) = (4, 2)$ to get $C' = 2 - \frac{171}{5} \ln 4$. So $\boxed{z = \frac{171}{5} \ln \left(\frac{x}{4}\right) + 2}$.

Chapter IV. PERFECT FLUID DYNAMICS

IV.1. Prerequisite Knowledge

Before studying this chapter, students should be familiar with:

- 💧 Newton's second law.
- 💧 Basic concept of mechanical energy (kinetic and potential energy).
- 💧 Manipulation of partial differential equations.
- 💧 Ability to interpret mathematical operators in a physical context.

IV.2. Learning Objectives

By the end of this chapter, the student should be able to:

- 💧 Describe the characteristics of flow
- 💧 Distinguish between the different flow regimes of a fluid.
- 💧 Explain the consequences of the equation of continuity to the conservation of mass.
- 💧 Apply Bernoulli's theorem for the steady flow of a real, incompressible fluid, with or without energy exchange.
- 💧 Describe the relationship between flow rate and velocity
- 💧 Determine the head losses and the power involved.

IV.3. Introduction & Context

In this chapter, the fluids studied have zero viscosity, and a constant density " ρ " (which can be the case for gases when their flow velocity is less than $0.3 \times$ the velocity of sound).

Two particularly important relationships for the adjuster will emerge from this chapter:

- 💧 The continuity relationship
- 💧 The Bernoulli equation

We will explore the differences between turbulent and laminar regimes in the following chapter. But the distinction is not necessary here, since the treated fluids are supposed to be perfect, that is to say that the fluid cells slide without friction on each other.

The different conservation principles of Physics will give us very important equations.

The principle of conservation:

- 💧 Of the MASS leads to the CONTINUITY EQUATION
- 💧 The QUANTITY OF MOVEMENT leads to EULER'S THEOREM
- 💧 ENERGY leads to BERNOULLI'S THEOREM

IV.4. Definition of Incompressible Perfect Fluid

An incompressible fluid is a fluid whose density remains constant under the application of external forces, such as pressure or temperature changes. (Panton, 2024). In fluid mechanics, the assumption of incompressibility simplifies fluid equations and makes the analysis more manageable for a wide range of engineering applications.

IV.5. Conservation of Mass

The conservation of mass relation for a closed system undergoing a change is expressed as:

$$m_{sys} = \text{constant} \text{ or } \frac{dm_{sys}}{dt} = 0, \quad \frac{\rho \cdot dV}{dt} = \rho \cdot \frac{dV}{dt} = \rho \cdot \frac{(dx \times dy \times dz)}{dt} = 0$$

which is the statement that the mass of the system remains constant during a process.

For a control volume (CV), mass balance is expressed in rate form as

The mass conservation at a volume element $dV = dx \times dy \times dz$ for steady, incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (\text{IV.1})$$

In general, the conservation of mass at a volume element may be formulated as follows (Oertel, 2004):

$$\begin{aligned} &\text{The rate of change of mass in a volume element} \\ &= \\ &\quad \sum \text{the mass fluxes into the volume element} \\ &\quad - \\ &\quad \sum \text{the mass fluxes out of the volume element} \end{aligned}$$

IV.6. Mass and Volume Flow Rates

IV.6.1. Mass Flow Rate

The first rate of change of interest for fluid in motion is the mass flow rate: the amount of mass that passes through a checkpoint in one unit time. Or, mathematically,

$$\text{Mass Flow Rate} = \dot{m} = \frac{\Delta m}{\Delta t}$$

However, since there may not always be a practically measurable amount of, say, water flowing through a pipe, calculating the mass of a fluid is difficult at times. Relations to density, $\rho = m/V$, are therefore preferred.

Combining this equation with the mass flow rate equation above gives:

$$\dot{m} = \frac{\Delta m}{\Delta t} = \rho \cdot \frac{\Delta V}{\Delta t} \quad (\text{IV.2})$$

Provided the pipe is **prismatic**, its volume can be expressed $V = A \cdot L$ where **A** is the cross-sectional area and **L** is a length perpendicular to that area.

$$\frac{\Delta m}{\Delta t} = \rho \cdot \frac{\Delta V}{\Delta t} = \rho \cdot \frac{A \cdot \Delta x}{\Delta t} = \rho \cdot A \cdot \frac{\Delta x}{\Delta t} = \rho \cdot v \cdot A \quad (\text{IV.3})$$

Therefore,

$$\text{Mass Flow Rate: } \dot{m} = \frac{\Delta m}{\Delta t} = \rho \cdot v \cdot A \quad (\text{IV.4})$$

IV.6.2. Volume flow rate

$$\text{Volume flow rate} = \frac{\Delta V}{\Delta t} = \frac{A \cdot \Delta L}{\Delta t} = A \frac{\Delta L}{\Delta t} = A \cdot v$$

Therefore,

$$\text{Volume Flow Rate: } Q_v = \frac{\Delta V}{\Delta t} = A \cdot v \quad (\text{IV.5})$$

$$\text{Mass Flow Rate: } \dot{m} = \rho \times \dot{V} \quad (\text{IV.6})$$

Be careful:

The mass flow is preserved, but the volume flow is only if the fluid is INCOMPRESSIBLE

Observation

In fluid flow analysis, four fundamental equations are employed to describe the general behavior of fluids: the **continuity equation**, momentum equation, energy equation, and equation of state.

However, when dealing with incompressible fluids where the density is constant, the energy equation becomes uncoupled and the equation of state simplifies to the statement that density is nearly constant. As a result, analyzing incompressible flow only necessitates continuity and momentum analysis.

IV.7. Continuity equation

The continuity equation describes the conservation of mass within a fluid flow. It states that the rate of mass entering a control volume must be equal to the rate of mass exiting the same volume, as shown in figure IV.2 below.

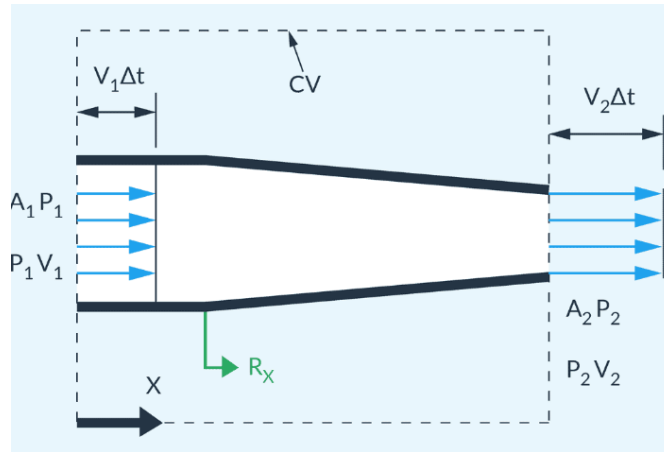


Fig.IV.2 Mass conservation in a control volume (Treiber & Kesting, 2025)

In general, the [continuity equation](#) for a steady, one-dimensional flow can be expressed as:

$$\frac{\Delta m_2}{\Delta t}_{out} = \frac{\Delta m_1}{\Delta t}_{in} \quad (IV.7)$$

According to the definition of [density](#) $m = \rho \cdot V$, so

$$\frac{\rho \Delta V_2}{\Delta t}_{out} = \frac{\rho \Delta V_1}{\Delta t}_{in} \quad (IV.8)$$

As long as the fluid is [incompressible](#), the density will not change from one section to the next, so ρ cancels out of both sides. Additionally, if the pipes are [prismatic](#) (as most conventional pipes are), then the volume can be expressed in terms of the cross-sectional area and length: $V = A \cdot L$. The equation is now

$$\begin{aligned} \frac{A \cdot \Delta L_2}{\Delta t}_{out} &= \frac{A \cdot \Delta L_1}{\Delta t}_{in} \\ A_2 \cdot \frac{\Delta L_2}{\Delta t}_{out} &= A_1 \cdot \frac{\Delta L_1}{\Delta t}_{in} \end{aligned}$$

Since $\frac{\Delta L}{\Delta t} = v$

$$A_2 \cdot v_2 = A_1 \cdot v_1 \quad (IV.9)$$

IV.8. Newton's Second Law

According to Newton's second law of motion, if the fluid motion is governed only by pressure and gravity forces it applies to a fluid particle in the form:

$$\begin{aligned}
 &(\text{Net pressure force on particle}) + (\text{Net gravity force on particle}) \\
 &= (\text{Particle mass}) \times (\text{Particle acceleration}) \\
 &\sum \vec{F}_{ext.} = m \cdot \vec{a}_G
 \end{aligned}
 \tag{IV.10}$$

We will apply this relation to a fluid particle, of very small dimensions

« dx », « dy », and « dz » and located at a coordinate point « x », « y » and « z ».

The volume of this particle is therefore $dV = dx \cdot dy \cdot dz$ (Pascal, 2020)

The volume containing this fluid particle is in the Earth's gravitational field (considered to be uniform):

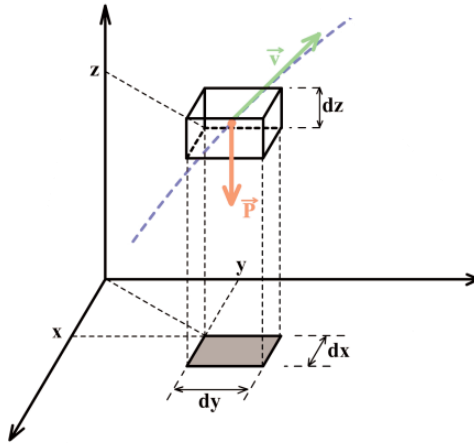


Fig.IV.3 Applied force on a fluid particle (Pascal, 2020)

In the case of a still fluid, the fundamental relation of dynamics allows us to obtain:

$$\frac{\partial p}{\partial x} \cdot dx = 0 ; \frac{\partial p}{\partial y} \cdot dy = 0 ; \frac{\partial p}{\partial z} \cdot dz = -\rho \cdot g \cdot dz \tag{IV.11}$$

The forces to take into account are the pressure forces exerted on the 6 faces of the cell, as well as the weight. The R.F.D. being a vector relationship, let's project this relationship onto each of the 3 axes.

💧 **Along the Ox axis: faces in « x » and in « x + dx »**

$$-p(x + dx, y, z) \cdot dy \cdot dz + p(x, y, z) \cdot dy \cdot dz = \rho \cdot dx \cdot dy \cdot dz \cdot \frac{dv_x}{dt} \tag{IV.12}$$

where v_x is the velocity of the cell according to Ox, and of course, $\rho \cdot dx \cdot dy \cdot dz$ is the mass of this cell.

Simplifying this equation gives:

$$-\frac{p(x + dx, y, z) - p(x, y, z)}{dx} = \rho \frac{dv_x}{dt} \tag{IV.13}$$

Or:

$$-\frac{\partial p}{\partial x} = \rho \frac{dv_x}{dt} \tag{IV.14}$$

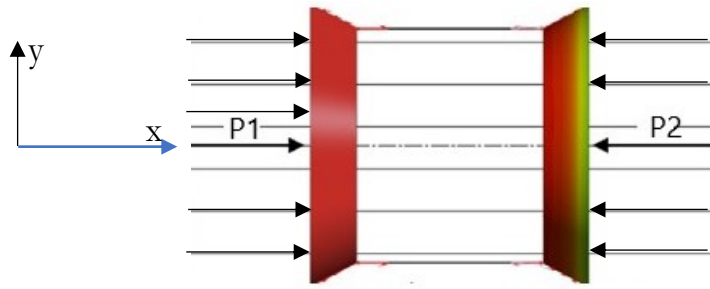


Fig.IV.4 Pressure force exerted on fluid surfaces.

🔥 Along the Oy axis: faces in « y » and in « y + dy »

$$- p(x; y + dy; z) \cdot dx \cdot dz + p(x, y, z) \cdot dx \cdot dz = \rho \cdot dx \cdot dy \cdot dz \cdot \frac{dv_y}{dt} \quad (\text{IV.15})$$

Or:
$$-\frac{\partial p}{\partial y} = \rho \frac{dv_y}{dt} \quad (\text{IV.16})$$

🔥 Along the Oz axis :

Here, the weight intervenes and the equation is written:

$$- \rho \cdot dx \cdot dy \cdot dz \cdot g - p(x, y, z + dz) \cdot dx \cdot dy + p(x, y, z) \cdot dx \cdot dy = \rho \cdot dx \cdot dy \cdot dz \cdot \frac{dv_z}{dt} \quad (\text{IV.17})$$

which gives:
$$-\rho \cdot g - \frac{\partial p}{\partial z} = \rho \frac{dv_z}{dt} \quad (\text{IV.18})$$

🔥 Expression of the total pressure differential:

The derivative of the pressure no longer makes sense here, which is why we introduced the notion of partial derivative (we drift in relation to only one of the variables, the others being considered as fixed...).

The impact of these small variations on the function is called the “differential” of the function and denoted by « d ».

So « dp » is the differential of the pressure function « p » and the link with the partial derivatives is:

$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz \quad (\text{IV.19})$$

We summarize this expression as follows:

- 🔥 $\frac{\partial p}{\partial x}$ is the rate of change of pressure in the direction « x » (same in « y » and « z »)
- 🔥 $\frac{\partial p}{\partial x} dx$ is therefore the value of the variation of the pressure in the direction « x » when the variable « x » varies by a small quantity « dx » (same in « y » et « z » ...)
- 🔥 this expression “simply” gives us the small variation « dp » which results from the small variations of its 3 variables (x, y and z).

Using the 3 equations (IV.14) (IV.16), and (IV.18):

$$dp = \left(-\rho \frac{dv_x}{dt}\right) \cdot dx + \left(-\rho \frac{dv_y}{dt}\right) \cdot dy + \left(-\rho \cdot g - \rho \frac{dv_z}{dt}\right) \cdot dz \quad (\text{IV.20})$$

Which can be written as: $dp = -\rho dv_x \frac{dx}{dt} - \rho dv_y \frac{dy}{dt} - \rho \cdot g \cdot dz - \rho \cdot dv_z \frac{dz}{dt}$

Either: $dp = -\rho \cdot v_x \cdot dv_x - \rho \cdot v_y \cdot dv_y - \rho \cdot g \cdot dz - \rho \cdot v_z \cdot dv_z$

Knowing that:

$$\vec{v} \begin{cases} v_x \\ v_y \\ v_z \end{cases} \text{ and } d\vec{v} \begin{cases} dv_x \\ dv_y \\ dv_z \end{cases}$$

$$\vec{v} \cdot d\vec{v} = v_x \cdot dv_x + v_y \cdot dv_y + v_z \cdot dv_z$$

So: $dp = -\rho \cdot v \cdot dv - \rho \cdot g \cdot dz$

$$\text{EULER Equation : } dp + \rho \cdot v \cdot dv + \rho \cdot g \cdot dz = 0 \quad (\text{IV.21})$$

This Euler equation has an immediate application: Bernoulli's theorem

IV.9. Bernoulli Theorem

The application of the principle of conservation of energy to frictionless laminar flow leads to a very useful relation between pressure and flow speed in a fluid. This relation is called Bernoulli's equation, named after Daniel Bernoulli (1700–1782), who published his studies on fluid motion in his book *Hydrodynamica* (1738). As the Figure below show, when a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. The increased kinetic energy comes from the net work done on the fluid to push it into the channel. Also, if the fluid changes vertical position, work is done on the fluid by the gravitational force.

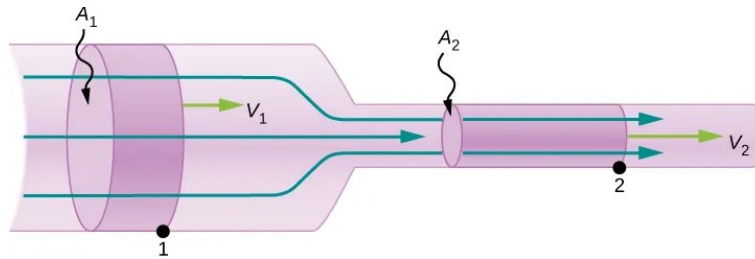


Fig.IV.5 kinetic energy on the fluid into the channel (Halliday et al., 2010).

A pressure difference occurs when the channel narrows. This pressure difference results in a net force on the fluid because the pressure times the area equals the force, and this net force does work. Recall the work-energy theorem.

$$w_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \quad (\text{IV.22})$$

The net work done increases the fluid's kinetic energy. As a result, the pressure drops in a rapidly moving fluid whether or not the fluid is confined to a tube.

IV.9.1. Energy Conservation and Bernoulli's Equation

Consider an incompressible fluid flowing through a pipe that has a varying diameter and height, as shown in Figure IV.6. Subscripts 1 and 2 in the figure denote two locations along the pipe and illustrate the relationships between the areas of the cross sections A , the velocity of flow v , the height from ground y , and the pressure p at each point.

We assume here that the density at the two points is the same. Therefore, density is denoted by without any subscripts and since the fluid is incompressible, the shaded volumes must be equal.

We also assume that there are no viscous forces in the fluid, so the energy of any part of the fluid will be conserved.

To derive Bernoulli's equation, we first calculate the work done on the fluid:

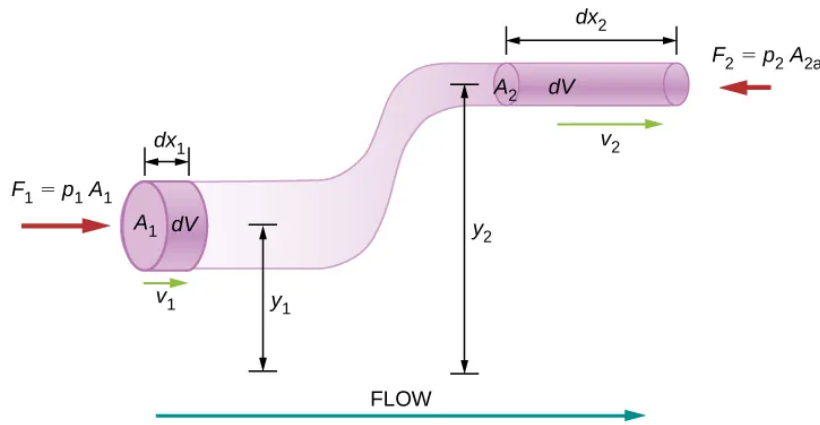


Fig.IV.6 Geometry used for the derivation of Bernoulli's equation (Halliday et al., 2010).

$$dW = F_1 dx_1 - F_2 dx_2 \quad (IV.23)$$

$$dW = p_1 A_1 dx_1 - p_2 A_2 dx_2 = p_1 dV - p_2 dV = (p_1 - p_2) dV \quad (IV.24)$$

The work done was due to the conservative force of gravity and the change in the kinetic energy of the fluid. The change in the kinetic energy of the fluid is equal to:

$$dK = \frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \rho dV (v_2^2 - v_1^2) \quad (IV.25)$$

The change in potential energy is:

$$dU = mgy_2 - mgy_1 = \rho \cdot dV \cdot g(y_2 - y_1) \quad (IV.26)$$

The energy equation then becomes:

$$dW = dK + dU \quad (\text{IV.27})$$

$$(p_1 - p_2)dV = \frac{1}{2}\rho dV(v_2^2 - v_1^2) + \rho \cdot dV \cdot g(y_2 - y_1) \quad (\text{IV.28})$$

$$(p_1 - p_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho \cdot g(y_2 - y_1) \quad (\text{IV.29})$$

Rearranging the equation gives Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho \cdot g \cdot z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho \cdot g \cdot z_2 \quad (\text{IV.30})$$

This relation states that the mechanical energy of any part of the fluid changes as a result of the work done by the fluid external to that part, due to varying pressure along the way. Since the two points were chosen arbitrarily, we can write Bernoulli's equation more generally as a conservation principle along the flow.

IV.9.2. Bernoulli's Equation

For an incompressible, frictionless fluid, the combination of pressure and the sum of kinetic and potential energy densities is constant not only over time, but also along a streamline:

$$p + \frac{1}{2}\rho v^2 + \rho \cdot g \cdot z = \text{Constant} \quad (\text{IV.31})$$

$$\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g(Z_2 - Z_1) = 0 \quad (\text{IV.32})$$

IV.9.2.1. Case of work exchange of fluid flow with a hydraulic machine

Hypotheses

Hydraulic machine placed between sections S1 and S2, characterized by a net power P_{net} exchanged with the fluid and a certain efficiency η .

This machine can be either a turbine or a pump.

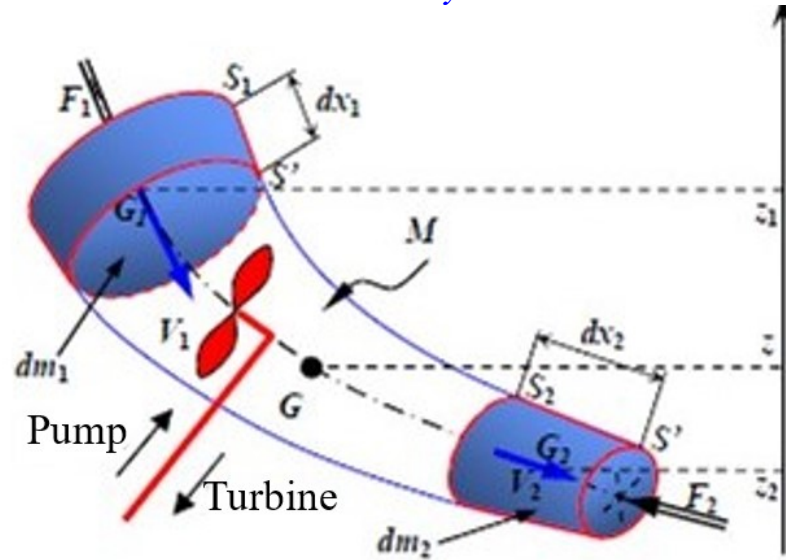


Fig.IV.7 Hydraulic machine between sections of a fluid vein(Hamouda, 2009)

- In the case of a pump: the efficiency is given by the following expression:

$$\eta = \frac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \frac{P_{net}}{P_{abs}} \quad (\text{IV.33})$$



- In the case of a turbine: the efficiency is the inverse of pump efficiency given by the following expression:

$$\eta = \frac{\text{Mechanical power output}}{\text{Mechanical energy removal from the fluid}} = \frac{P_{abs}}{P_{net}} \quad (\text{IV.34})$$



When fluid passes through a hydraulic machine, it exchanges energy with that machine.

We get the following Bernoulli equation:

$$\frac{v_2^2 - v_1^2}{2} + \frac{P_2 - P_1}{\rho} + g(Z_2 - Z_1) = \frac{P_{net}}{\dot{m}} \quad (\text{IV.35})$$

IV.10. hydraulic head

In general, the hydraulic head, or total head, is a measure of the potential of fluid at the measurement point. It can be used to determine a hydraulic gradient between two or more points.

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho \cdot g \cdot h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho \cdot g \cdot h_2 \quad (\text{IV.36})$$

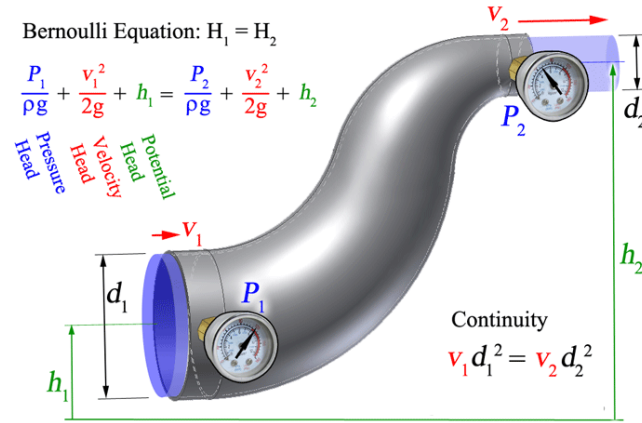


Fig.IV.8 Hydraulic head, of fluid at the measurement point(Sultanian, 2021).

IV.10.1. Bernoulli's principle and Venturi effect

Bernoulli's principle states that as the speed of a fluid increases, the pressure and temperature decrease. The venturi effect is the demonstration of this principle in a confined pipe.

$$Q_v = A_2 \cdot v_2 = A_1 \cdot v_1 \quad (\text{IV.37})$$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \text{ as } h_1 = h_2$$

$$p_1 - p_2 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 0 \quad (\text{IV.38})$$

Therefore: **Venturi effect**

$$Q_v = A_1 \sqrt{\frac{2(p_1 - p_2)}{\rho \left(\left(\frac{A_1}{A_2} \right)^2 - 1 \right)}} \quad (\text{IV.39})$$

The Venturi effect is a fluid dynamics phenomenon where a fluid's pressure decreases as it flows through a constricted section of a pipe, leading to an increase in its velocity (Qin & Duan, 2017).

The Venturi effect occurs when a fluid moves through a pipe that narrows in the middle and then widens again as shown below:

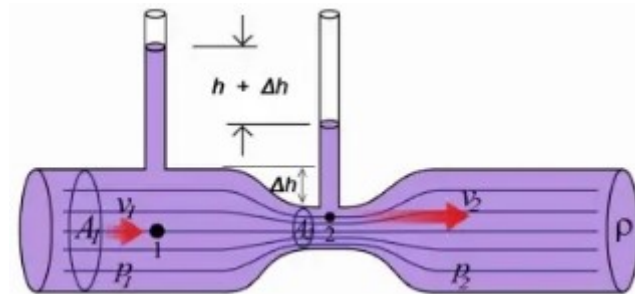


Fig.IV.9 Venturi principle (Sultanian, 2021)

IV.10.2. Applications of the Venturi Effect

The Venturi effect has numerous practical applications across various fields, including (Monchwe, 2023):

- 💧 **Flow Measurement:** Devices like Venturi meters utilize the pressure difference created by the Venturi effect to measure the flow rate of fluids in pipes.



Fig.IV.10 Venturi Meters in Industrial Applications

- 💧 **Carburetors:** In automotive engineering, carburetors use the Venturi effect to mix air and fuel in internal combustion engines, ensuring efficient combustion. The principle of the carburetor is based on the Bernoulli principle. The carburetor allows the gasoline and air to enter the engine's

combustion chamber as the pressure decrease and velocity increase. The venturi tube, the primary component of any carburetor, is solely responsible for its operation.

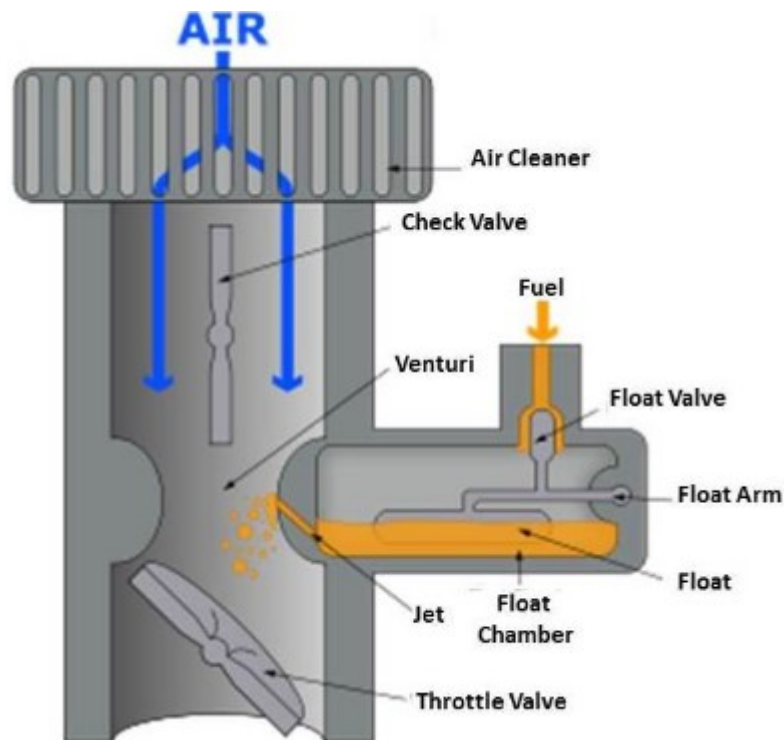


Fig.IV.11 Venturi effect carburetor (Nawale, 2021)

- 💧 **Aspirators:** By using the Venturi effect, that generates a pressure drop at the constriction and permits the entrainment of fluids or gases, these devices generate a vacuum.

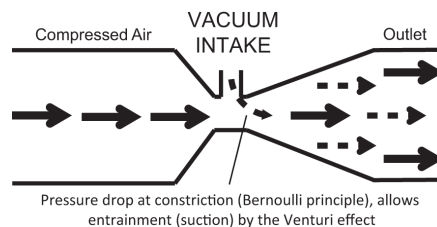


Fig.IV.12 Venturi Effect Vacuum

- 💧 **Irrigation Systems:** The effect is also used in irrigation systems to control and measure the flow of water. A vacuum is created the water flows through a converging passage that gradually widens.

Injection is activated when there is a pressure differential between the water entering the injector and the water and chemical leaving the irrigation system.

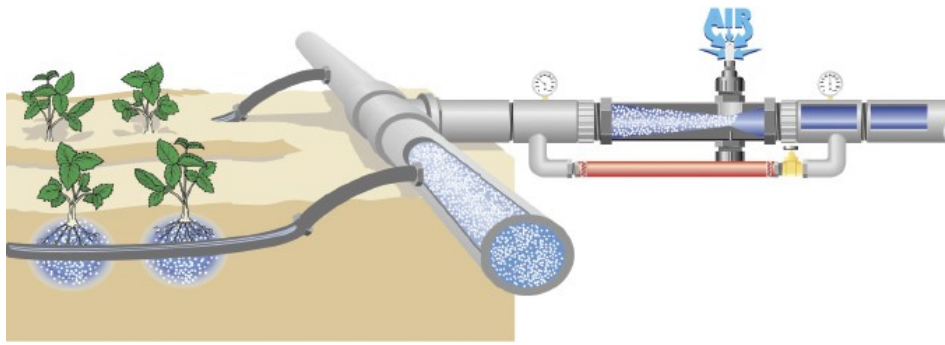


Fig.IV.13 Venturi injector (Baylar & Ozkan, 2006)

- 💧 **Aerodynamics:** Understanding airflow over wings and through turbines is enhanced by the principles of the Venturi effect, which is crucial for designing efficient aerodynamic structures. A Venturi tube is installed on the side of the fuselage. The tube inlet creates a suction, allowing wind to pass through the tube. As the air passes through the central constriction, the air molecules speed up to facilitate the passing of the same amount of air molecules from the outlet as entered through the inlet. This ensures the conservation of mass for incompressible, inviscid fluid. Once passed through the constriction, the air molecules slow down again during the exit. The suction created by the Venturi tube in airplanes can play an important role in operating gyroscopic instruments. In simple early aircraft, the Venturi tube provides a low-cost option for creating suction.

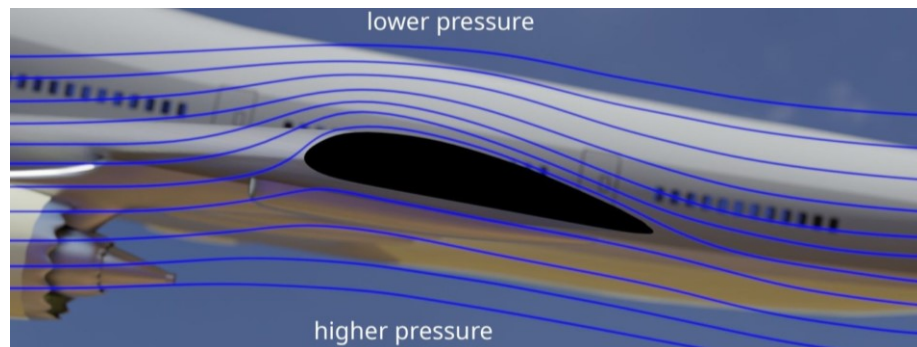


Fig.IV.14 Aircraft winds (Zhang, 2025)

IV.11. Static, Stagnation, Dynamic, and Total Pressure

The simplified form of Bernoulli's equation can be summarized in the following memorable word equation

$$\text{Static pressure} + \text{dynamic pressure} = \text{total pressure (stagnation pressure)}$$

Total and dynamic pressure are not pressures in the common sense. It can be only measure by putting the fluid to rest, e.g. using a pitot tube. That's why it's sometimes called **stagnation pressure**.

IV.11.1. Static Pressure

In fluid dynamics, many authors use the term **static pressure** in preference to just **pressure** to avoid ambiguity. The term **static pressure** is identical to the term **pressure** and can be identified for every point in a fluid flow field.

Bernoulli's effect causes the lowering of fluid pressure (**static pressure**) in regions where the flow velocity increases.

IV.11.2. Dynamic pressure

Dynamic pressure is one of the terms of Bernoulli's equation. In incompressible fluid dynamics, dynamic pressure is the quantity defined by:

$$p_{dyn} = \frac{1}{2} \rho v^2 \quad (\text{IV.40})$$

IV.11.3. Stagnation pressure (total pressure)

In fluid dynamics and aerodynamics, **stagnation pressure** (or **pitot pressure** or **total pressure**) is the static pressure at a stagnation point in a fluid flow.

At a stagnation point, the fluid velocity is zero, and all **kinetic energy** has been converted into **pressure energy** (isentropically).

This effect is widely used in aerodynamics (velocity measurement or ram-air intake).

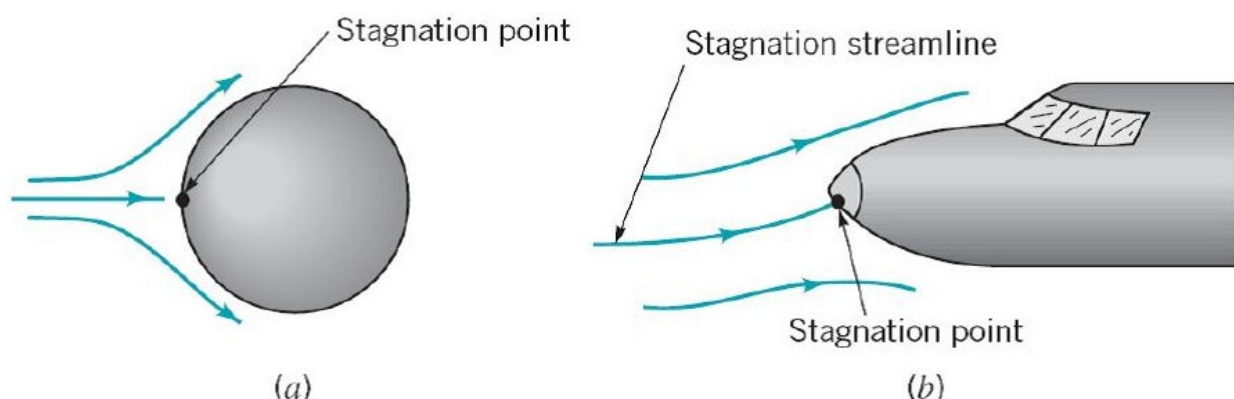
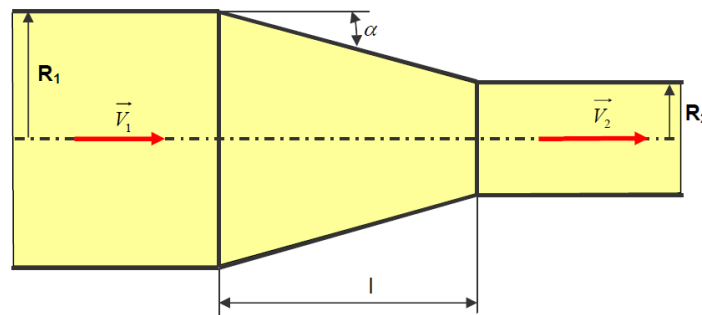


Fig.IV.15 Stagnation points on bodies in flowing fluids (Vogel, 1996)

IV.12. Problem Set

Exercise N°1

To accelerate the flow of a perfect fluid in a pipe so that its speed is multiplied by 4. To achieve this, the pipe has a convergent section characterized by the angle α .



- 1) Calculate the ratio of the radius (R_1/R_2).
- 2) Calculate ($R_1 - R_2$) as a function of L and α . Deduce the length L . ($R_1 = 50 \text{ mm}$, $\alpha = 15^\circ$).

Solution

- 1) Let's apply continuity equation to the flow:

$$A_2 \cdot v_2 = A_1 \cdot v_1 \text{ or } \frac{A_1}{A_2} = \frac{v_2}{v_1}; S_1 = \pi R_1^2 \text{ and } S_2 = \pi R_2^2 \text{ so } \frac{R_1}{R_2} = \sqrt{\frac{v_2}{v_1}} = 2$$

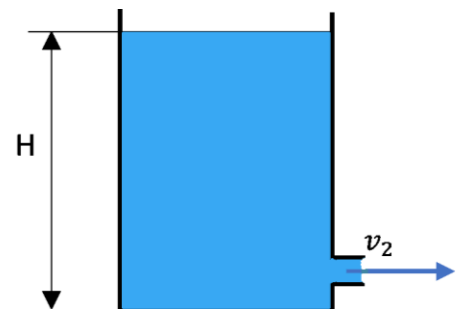
$$2) \text{tg } \alpha = \frac{R_1 - R_2}{l} \text{ so } l = \frac{R_1 - R_2}{\text{tg } \alpha} = 93.3 \text{ mm}$$

Exercise N°2

Consider a reservoir filled with water to a height $H = 3 \text{ m}$, equipped with a small orifice at its base with a diameter $d = 10 \text{ mm}$.

- 1) Specifying the assumptions made, apply Bernoulli's theorem to calculate the water flow velocity v_2 .

- 2) Deduce the volumetric flow rate $\dot{V}$ in (l/s) at the outlet of the orifice.



Solution

- 1) Let's apply Bernoulli's theorem between the up surface of the reservoir ❶ and the outlet of the orifice ❷.

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho \cdot g \cdot z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho \cdot g \cdot z_2$$

$$p_1 = p_2 = p_{atm}$$

$$z_1 - z_2 = H$$

$$v_1 \text{ is negligible to } v_2, \text{ so } v_1 \approx 0$$

$$\frac{1}{2}(v_2^2 - v_1^2) = g \cdot H$$

$$v_2 = \sqrt{2gH} = 7.66 \text{ m/s}$$

$$2) \text{ The volumetric flow rate } Q_v = A_2 \cdot v_2 = \frac{\pi(d_2)^2}{4} \cdot v_2 = 0.000601 \text{ m}^3/\text{s} = 0.601 \text{ l/s}$$

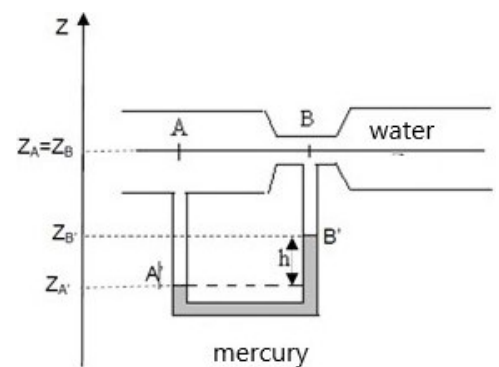
Exercise N°3

In a horizontal pipe with a diameter $D = 9 \text{ cm}$, we want to measure the water flow rate. A Venturi tube ($D = 9 \text{ cm}$, $d = 3 \text{ cm}$) is inserted. The difference in height h of the mercury in a U-shaped tube can be measured accurately.

Given: $\rho_{\text{mercury}} = 13600 \text{ kg/m}^3$,

Required work:

- 1) Deduce the average flow velocity v_B at the throat in section S_B as a function of the velocity v_A in section S_A .
- 2) Determine the pressure difference: $(P_{A'} - P_{B'})$ as a function of g , ρ_{mercury} , $Z_{A'}$, and $Z_{B'}$.
- 3) Similarly, determine the expression for the pressure difference $(P_A - P_{A'})$ as a function of g , ρ_{water} , $Z_{A'}$, and Z_A .
- 4) Likewise, determine the expression for the pressure difference $(P_{B'} - P_B)$ as a function of g , ρ_{water} , $Z_{B'}$, and Z_B .
- 5) Assuming that water is a perfect fluid, and applying Bernoulli's theorem between A and B, give the expression for the flow velocity v_A as a function of the pressure difference $(P_A - P_B)$, ρ_{water} .
- 6) Deduce the expression for the volumetric flow rate Q_v as a function of D , ρ_{mercury} , ρ_{water} , g , h .
- 7) Perform a numerical application for a difference in elevation $h = 4 \text{ mm}$.



Solution

- 1) Let's applied continuity equation: $S_A \cdot v_A = S_B \cdot v_B$ so or $v_B = \frac{S_A}{S_B} v_A = 9v_A$.
- 2) hydrostatic theorem between A' and B' gives: $p_{A'} - p_{B'} = \rho_{mercury} g (Z_{B'} - Z_{A'})$
- 3) hydrostatic theorem between A and A' gives: $p_A - p_{A'} = \rho_{water} g (Z_{A'} - Z_A)$
- 4) hydrostatic theorem between B' and B gives: $p_{B'} - p_B = \rho_{water} g (Z_B - Z_{B'})$
- 5) by summing the previous relationships:

$$p_A - p_B = (\rho_{mercury} - \rho_{water}) \cdot g \cdot (Z_B - Z_{B'}) = (\rho_{mercury} - \rho_{water}) \cdot g \cdot h$$

- 6) Bernoulli's theorem between A and B gives: $\frac{v_A^2 - v_B^2}{2} + \frac{P_A - P_B}{\rho} + g(Z_A - Z_B) = 0$

$$\text{But } Z_A = Z_B \text{ and } v_B = 9v_A \text{ so } v_A = \sqrt{\frac{(\rho_{mercury} - \rho_{water}) \cdot g \cdot h}{40 \cdot \rho_{water}}}$$

- 7) Flow rate: $Q_v = S_A \cdot v_A = \frac{\pi D^2}{4} \sqrt{\frac{(\rho_{mercury} - \rho_{water}) \cdot g \cdot h}{40 \cdot \rho_{water}}}$

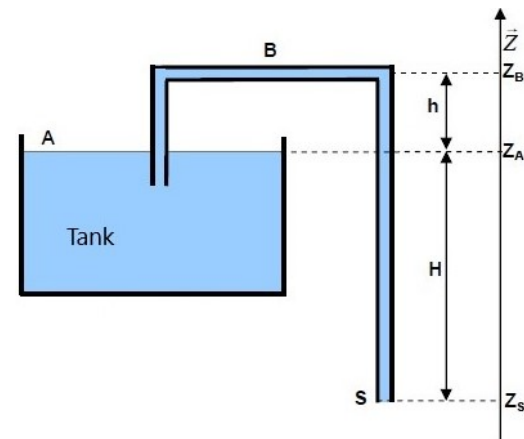
- 8) Numerical application: $Q_v = 7 \cdot 10^{-4} \text{ m}^3/\text{s}$

Exercise N°4

Consider a siphon with a diameter $d = 10 \text{ mm}$, fed by a gasoline tank that is large compared to d and open to the atmosphere.

Assume that: the density of gasoline: $\varpi = 6896 \text{ N/m}^3$, $H = 2.5 \text{ m}$.

- 1) Calculate the flow velocity v_s in the siphon.
- 2) Deduce the volumetric flow rate Q_v .
- 3) Give the expression for the pressure p_B at point B as a function of h , H , ϖ , and P_{atm} .
- 4) Perform a numerical calculation for $h = 0.4 \text{ m}$.
- 5) Can "h" take any value? Justify your answer.



Solution

1) Bernoulli's theorem between A and B gives: $\frac{v_S^2}{2g} + \frac{P_S}{\varpi} + Z_S = \frac{v_A^2}{2g} + \frac{P_A}{\varpi} + Z_A$

we know that $p_A = p_S = p_{atm}$, $v_A = 0$ and $Z_A - Z_S = H$; So $v_S = \sqrt{2 \cdot g \cdot H} = 7 \text{ m/s}$

2) Flow rate: $Q_v = v_S \cdot \frac{\pi D^2}{4} = 5.5 \cdot 10^{-4} \text{ m}^3/\text{s} = 0.55 \text{ l/s}$

3) Bernoulli's theorem between B and S gives: $P_B = P_{atm} - \varpi \cdot (H + h)$; $p_B = 80001.6 \text{ Pa} = 0.8 \text{ bar}$

4) No. p_B must be greater than 0 ($p_B > 0$), so $\varpi \cdot (H + h) < P_{atm}$ and $h < \frac{P_{atm}}{\varpi} + H$ which means that

$$h < 12 \text{ m}$$

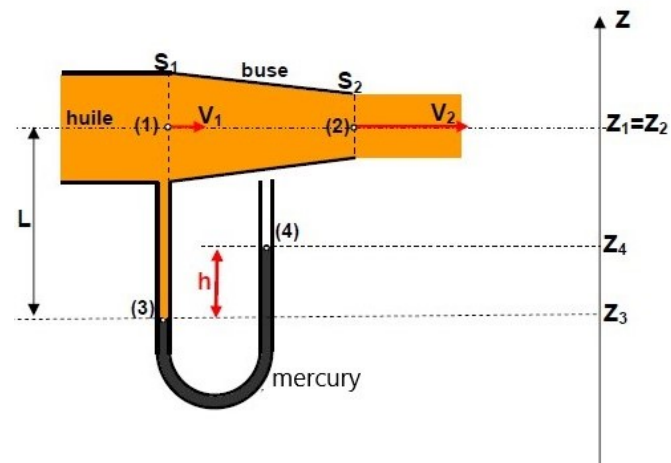
Exercise N°5

Oil is accelerated through a converging cone-shaped nozzle.

The nozzle is equipped with a U-shaped manometer containing mercury.

Part 1: Nozzle Analysis

With a volumetric flow rate $Q_v = 0.4 \text{ L/s}$, the oil flows through section S_1 with a diameter $d_1 = 10 \text{ mm}$ at a flow velocity v_1 and a pressure p_1 , exiting into the atmosphere through section S_2 with a diameter d_2 at a flow velocity $v_2 = 20 \text{ m/s}$ and a pressure $p_2 = P_{atm} = 1 \text{ bar}$, $\rho_{oil} = 800 \text{ kg/m}^3$.



1) Calculate the flow velocity v_1 .

2) Deduce the diameter d_2 .

3) Determine the pressure p_2 in bar.

Part 2: Study of the Manometer (U-tube).

The manometer, a U-tube, contains mercury. It allows the pressure p_1 to be measured from a reading of the height difference "h". Given: $L = 1274 \text{ mm}$.

1) Determine the pressure p_3 .

2) Similarly, determine the height difference "h" of the mercury.

Part 1: Nozzle Analysis

- 1) flow velocity $v_1 = \frac{4\dot{V}}{\pi \cdot d^4} = 5 \text{ m/s}$
- 2) continuity equation: $S_1 \cdot v_1 = S_2 \cdot v_2$ or $d_2 = \sqrt{\frac{v_1}{v_2}} d_1 = 5 \text{ mm}$.
- 3) Bernoulli's theorem gives: $\frac{v_2^2 - v_1^2}{2} + \frac{P_2 - P_1}{\rho_{oil}} + g(Z_2 - Z_1) = 0$ but $Z_1 = Z_2$ and $p_2 = p_{atm}$
so, we have: $p_1 = p_2 + \frac{1}{2} \rho_{oil} (v_2^2 - v_1^2) = 2.5 \cdot 10^5 \text{ Pa} = 2.5 \text{ bar}$

Part 2: U-tube

- 4) Applied the RFH between (1) and (3): $p_3 = p_1 + \rho_{oil} \cdot g \cdot L = 2.6 \cdot 10^5 \text{ Pa} = 2.6 \text{ bar}$
- 5) Similarly, Applied the RFH between (3) and (4): $p_3 - p_4 + \rho_{mercury} \cdot g \cdot (Z_4 - Z_3)$

knowing that: $Z_4 = Z_3 = h$

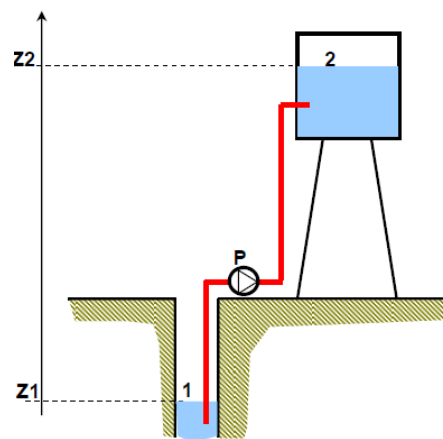
$$\text{So } h = \frac{p_3 - p_4}{\rho_{mercury} \cdot g} = 1.2 \text{ m}$$

Exercise N°6

A pump P supplies a water tower from a well through a pipe with a diameter $d = 150 \text{ mm}$.

Given: $Z_2 = 26 \text{ m}$, $Z_1 = -5 \text{ m}$, $p_1 = p_2 = 1.013 \text{ bar}$; $v = 0.4 \text{ m/s}$,

- 1) Calculate the volumetric flow rate Q_v of the pump in l/s.
- 2) Write Bernoulli's equation between surfaces 1 and 2.
- 3) Calculate the useful power P_{net} of the pump.
- 4) Given that the pump's efficiency is 80%, calculate the power P_a that it absorbs



Solution

1. Flow rate: $Q_v = v \cdot \frac{\pi d^2}{4} = 7 \cdot 10^{-3} \text{ m}^3/\text{s} = 0.55 \text{ l/s} = 7 \text{ l/s}$
2. Bernoulli equation for an incompressible flow with machine work exchange:

$$\frac{v_2^2 - v_1^2}{2} + \frac{P_2 - P_1}{\rho} + g(Z_2 - Z_1) = \frac{P_{net}}{\rho \cdot Q_v}$$

3. Useful power P_{net} of the pump:

$$P_{net} = Q_v \cdot \rho \cdot g \cdot (H_1 + H_2) = 7 \cdot 10^{-3} \cdot 1000 \cdot 9.81 \cdot (26 + 5) = 2128.77 \text{ W}$$

4. Absorbed power: $P_a = \frac{P_{net}}{\eta} = 2661 \text{ W}$

Chapter V. REAL FLUID DYNAMICS

V.1. Learning Objectives

- 💧 Understand the physical meaning of viscosity
- 💧 Distinguish between laminar and turbulent flows
- 💧 Use the Navier–Stokes equations for viscous flows
- 💧 Compute pressure losses in pipes (major + minor losses)
- 💧 Apply the Reynolds number and friction factor in simple systems
- 💧 Understand boundary layer development and separation

V.2. Prerequisite Knowledge

Students should already master:

V.2.1. Mathematics Prerequisites

- 💧 Vector & Calculus
- 💧 Differential Equations in Fluid Mechanics
- 💧 Basic Tensor Concepts

V.2.2. Physics Prerequisites

- 💧 Fluid Statics
- 💧 General Fluid Properties
- 💧 Perfect Fluid Dynamics

V.3. Introduction & Context

This course covers the study of **real fluids**, that is, fluids with **viscosity**. We see how Euler's equation and **Bernoulli's** relation are modified to account for dissipative effects within the fluid. We also introduce the concepts of **pressure drop** and aerodynamic coefficients.

The ideal fluid studied in the previous chapter does not always accurately reflect reality. Fluids all have varying degrees of viscosity, which must be taken into account, particularly **near the walls** of solids or in the areas they leave behind.

To solve a real fluid flow problem, **experimental results** are used. A simplified method for calculating **head losses** based on these experimental results is proposed. It is essential for sizing various hydraulic installations (pumping, turbines, hydraulic and thermal machines in which a real fluid is conveyed, etc.).

It is known that in real fluid flow, fluid particles are subjected to **frictional** forces due to viscosity and **turbulence** during their movement. To overcome these forces, kinetic energy is dissipated and transformed into thermal energy, resulting in a **head loss** (a pressure drop).

V.4. Definition

Real flow describes a fluid's movement where its **internal friction**, or **viscosity**, plays a role. This internal resistance to flow distinguishes real-world fluids from theoretical “**perfect**” fluids that are considered frictionless (Papanastasiou et al., 2021b). A simple way to visualize this is by comparing how water and honey pour from a container. Water, with low viscosity, flows quickly, while honey, with high viscosity, flows slowly and resists movement.

V.5. Ideal Fluid vs. Real Fluid: Key Differences

When we talk about fluids, whether it's the water we drink or the air we breathe, we often simplify things in physics to make calculations easier. This leads us to the concept of an “ideal fluid.” But how does an ideal fluid compare to the real fluids we encounter every day?

V.5.1. What is an Ideal Fluid?

An ideal fluid is a theoretical concept. We define it as a fluid that is both incompressible and non-viscous. This means a few key things (Kiselev et al., 1999)

- 💧 **Incompressible:** Its volume doesn't change under pressure. Think of trying to squeeze a balloon filled with water – it's really hard! Ideal fluids behave similarly.
- 💧 **Non-viscous:** It has no internal friction. Imagine pouring honey versus water; honey is more viscous. An ideal fluid would flow without any of this resistance.
- 💧 **Infinite Bulk Modulus:** This means it takes an infinite amount of pressure to compress it.
- 💧 **No Surface Tension:** Surface tension is what allows water droplets to form. Ideal fluids don't have this property.
- 💧 **No Resistance to Shear or Tensile Forces:** Ideal fluids can't resist internal forces that try to deform or pull them apart.

It's important to note: **Ideal fluids are imaginary and don't exist in nature.** They are simply a useful simplification for theoretical calculations. They exhibit smooth, **laminar flow**, which is depicted below:



Fig.V.1 Typical velocity profile (a) Ideal fluid (b) Real fluid(Sachin Thorat, n.d.)

V.5.2. What is a Real Fluid?

Now, let's talk about fluids we actually see in the real world. Real fluids are, well, real.

They possess properties that ideal fluids lack:

- 💧 **Compressible:** Their volume can change under pressure. While liquids are not as easily compressible as gases, they are still compressible to some extent.
- 💧 **Viscous:** They have internal friction, meaning they resist flow. This is why some fluids move more slowly than others.
- 💧 **Surface Tension:** Real fluids exhibit surface tension.
- 💧 **Friction Effects:** Real fluids experience friction as they move and interact with surfaces.
- 💧 **Turbulent Flow:** Real fluids often exhibit turbulent flow, which is chaotic and less predictable, unlike the smooth flow of ideal fluids.

Examples of real fluids are everywhere: kerosene, petrol, and even castor oil.

Here's a table to clearly outline the differences between ideal and real fluids:

Table V.1 Characteristic of ideal and real fluids

Characteristic	Ideal fluid	Real fluid
Compressible	No	Yes
Viscosity	No	Yes
Surface Tension	No	Yes
Flow Type	Laminar	Turbulent
Friction	No	Yes

In essence, ideal fluids are a useful theoretical construct that helps simplify fluid dynamics calculations. However, it is important to remember that real fluids have characteristics like viscosity and compressibility, which affect their behavior.

V.6. Viscosity

The viscosity is a physical quantity that describes a fluid's resistance to flow (Westerhof et al., 2019). Viscosity denotes opposition to flow.

A liquid's viscosity is a measure of its resistance to gradual deformation. For liquids, it corresponds to the informal notion of "thickness". For example, honey has a much higher viscosity than water.

V.6.1. Dynamic viscosity

A force is required to move F at a constant velocity $\Rightarrow$ there is a dissipative process. It's defined as the resistance to movement of one layer of a fluid over another as shown in figure V.1.



Fig.V.2 Shear stress Vs velocity gradient

Since viscosity measures the resistance of a fluid deformed by shear, it is expressed as the ratio of the shear stress to the velocity gradient:

$$F = -\mu A \frac{du}{dy} \quad (V.1)$$

Where:

F is the applied force

A is the area over which the force is applied

$\partial u / \partial y$ is the velocity gradient

μ is the dynamic viscosity $[\text{N} \cdot \text{m}^{-2} \cdot \text{s}] = [\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}]$,

For practical applications, smaller units are often used. The **centipoise (cP)**, from the CGS system, is commonly employed, where $1 \text{ cP} = 0.001 \text{ Pa} \cdot \text{s} = 0.001 \text{ N} \cdot \text{s} / \text{m} = 0.01 \text{ g} \cdot \text{cm}^{-1} \cdot \text{s}^{-1}$. Water at room temperature (20°C) has a dynamic viscosity of approximately 1 cP, making it a convenient reference.

Table V.2. Typical value of the viscosity

Liquid	Absolute Viscosity (N s/m ² , Pa s)
Air	1.983×10^{-5}
Water	10^{-3}
Olive Oil	10^{-1}
Glycerol	10^0
Honey	10^1
Golden Syrup	10^2
Glass	10^{40}

V.6.2. Kinematic viscosity

Kinematic viscosity is the ratio of - absolute (or dynamic) viscosity to density - a quantity in which no force is involved. Kinematic viscosity can be obtained by dividing the absolute viscosity of a fluid with the fluid mass density like

$$\gamma = \frac{\mu}{\rho} [m^2 / s] \quad (V.2)$$

Where:

γ : kinematic viscosity (m²/s)

μ : absolute or dynamic viscosity (N s/m²)

ρ : density (kg/m³)

In the SI-system the theoretical unit of kinematic viscosity is **m²/s** or the commonly used **Stoke (St)** where:
1 St (Stokes) = 10⁻⁴ m²/s = 1 cm²/s

V.7. Types of flow

All fluid flow is classified into one of two broad categories or regimes. These two flow regimes are laminar flow and turbulent flow.

Understanding the characteristics and implications of these flow patterns is essential for predicting and controlling fluid behavior in engineering applications.

V.7.1. Laminar Flow

Laminar flow is referred to as streamline or viscous flow. It occurs when a fluid flows smoothly in parallel layers without any significant mixing.

In practical applications:

- Fluid flow is smooth and ordered.
- Frictional losses are minimized.
- Noise and vibrations are reduced.

V.7.2. Turbulent Flow

Turbulent flow is characterized by the irregular movement of particles of the fluid.

The particles travel in irregular paths with no observable pattern and no definite layers.

In practical applications:

- Fluid flow is chaotic and disordered.
- Friction losses are increased.
- Noise and vibrations may be significant.

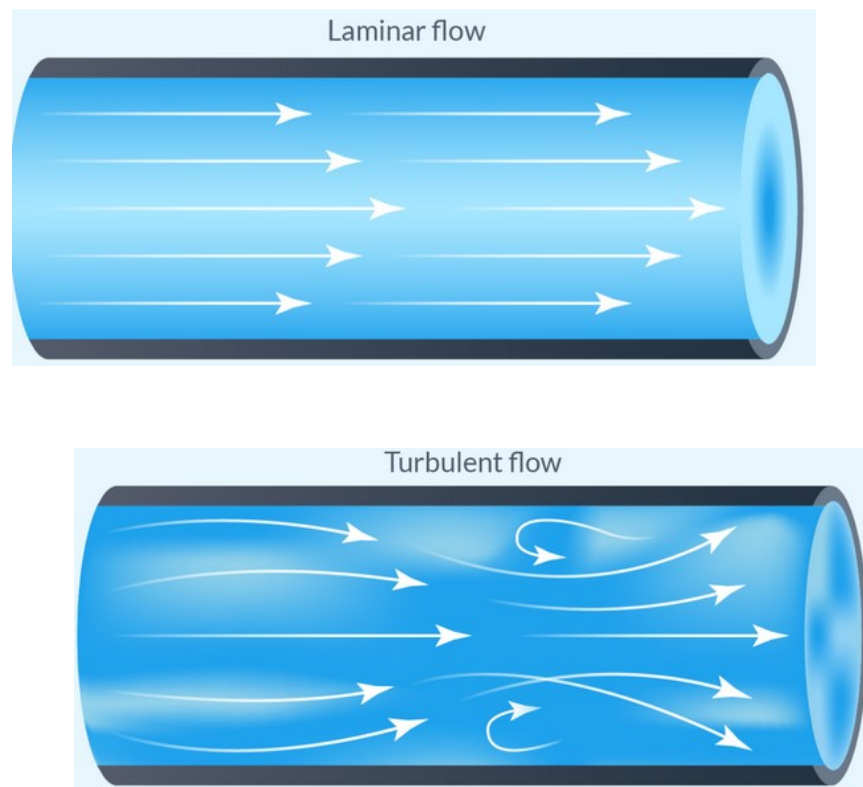


Fig.V.3 Flow Velocity Profiles

V.7.3. Reynolds Number and Flow Pattern

Reynolds number is an important dimensionless parameter that identifies the behavior of fluid based on attributes like viscosity or velocity of the fluid. Accordingly, the value of Reynolds number (N_R) can be expressed as:

$$N_R = \frac{\rho u D_{hyd}}{\mu} = \frac{\rho u (2R)}{\mu} \quad (V.3)$$

Note that:

ρ is the density of the fluid

u is the axial fluid velocity

D_{hyd} is the hydraulic diameter (of pipe, tube, or duct).

For practical purposes, if the Reynolds number is:

- 💧 Laminar if $N_R < 2000$
- 💧 Turbulent if $N_R > 3500$
- 💧 Transitional flows if $2000 < N_R < 3500$

Table V.3 : A spectrum of Reynolds numbers (Grant et al., 1959)

Type of flow	Reynolds Numbers
A large whale swimming at 10 m/s	300.000.000
A tuna swimming at 10 m/s	30.000.000
A duck flying at 20 m/s	300. 000
Flapping wings of the smallest flying insects	30
Atmospheric flow	10^{11}
Blood flow in the aorta	10^4
Field of aerodynamics	10^7
Microfluidics field	10^{-3} to 1

V.8. Hydraulic Diameter

For flow in non-circular pipes, the Reynolds number is based on the hydraulic diameter D_{hyd} defined as:

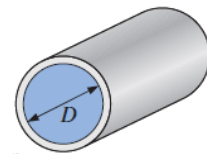
$$D_{hyd} = \frac{4A_c}{p} \quad (V.4)$$

Where A_c is the cross-sectional area of the pipe and p is its wetted perimeter.

The hydraulic diameter is defined such that it reduces to the ordinary diameter.

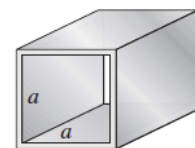
Circular Conduct

$$D_{hyd} = \frac{4 \left(\frac{\pi D^2}{4} \right)}{\pi D} = D$$



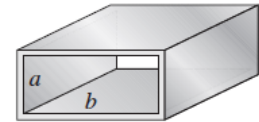
Square Conduct

$$D_{hyd} = \frac{4a^2}{4a} = a$$



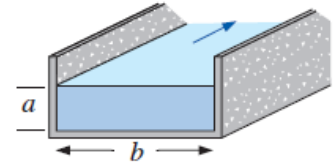
Rectangular Conduct

$$D_{hyd} = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$$



Channel

$$D_{hyd} = \frac{4ab}{2a+b}$$



V.9. Pressure Drop

The pressure drop ΔP is a decrease in pressure as fluid flows through a system, usually due to friction and obstructions.

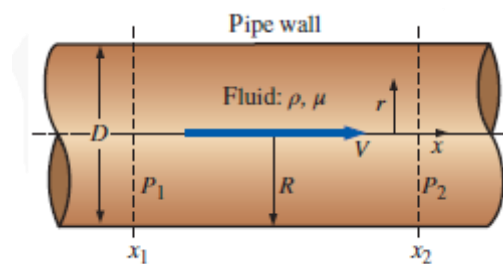


Fig.V.4 Pressure drop in a Steady laminar flow in a long round pipe (Çengel & Cimbala, 2019)

For all kinds of fully developed internal flows (laminar or turbulent flows, circular or non-circular pipes, smooth or rough surfaces, horizontal or inclined pipes), it is practical to express the pressure drop as follows:

$$\Delta P = f \frac{L}{D} \rho \frac{v_{avg}^2}{2} \quad (V.6)$$

where $\rho \frac{v_{avg}^2}{2}$ is the dynamic pressure and f is the Darcy friction factor,

$$f = \frac{8\tau_w}{\rho v_{avg}^2} \quad (V.7)$$

τ_w is the shear stress force

The pressure drops $\nabla P = p_1 - p_2$ between two points (1) and (2) in a hydraulic circuit (fig. V.4) originates from:

- 💧 Friction of the fluid on the inner wall of the pipe: these are called regular or systematic pressure losses.
- 💧 Resistance to flow caused by flow disturbances (bends, enlargements or reductions in cross-section, control devices, etc.): these are called accidental or singular pressure losses.

The calculation of its pressure drops depends on:

- 💧 Fluid characteristics: density, kinematic viscosity
- 💧 Pipe characteristics: shape and dimensions, length, roughness

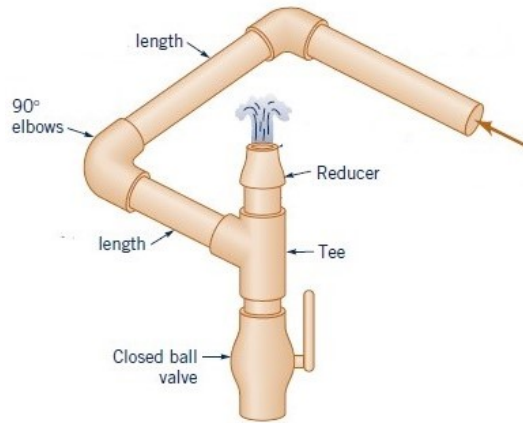


Fig.V.5 Hydraulic circuit

The value of the major loss coefficient f depends on:

- 💧 In a laminar flow regime: $f = \frac{64}{Re}$
- 💧 In a smooth turbulent flow regime for $Re < 10^5$: Blasius Formula; $f = 0.316 Re^{-0.25}$
- 💧 In a rough turbulent flow regime for $Re > 10^5$: Blench formula; $f = 0.79 \sqrt{\frac{\epsilon}{D}}$

With ϵ : roughness of the internal surface of the pipe (mm). In practice (Jiang et al., 2025):

welded steel pipes: $\epsilon \in [0.15; 0.20]$

Galvanized iron: $\epsilon \in [0.015; 0.020]$

Copper: $\epsilon \approx 0.000015$

V.10. Moody Diagram

This is a nomogram for directly calculating the pressure loss coefficient, based on the Reynolds number and the relative roughness of the internal wall of the pipe.

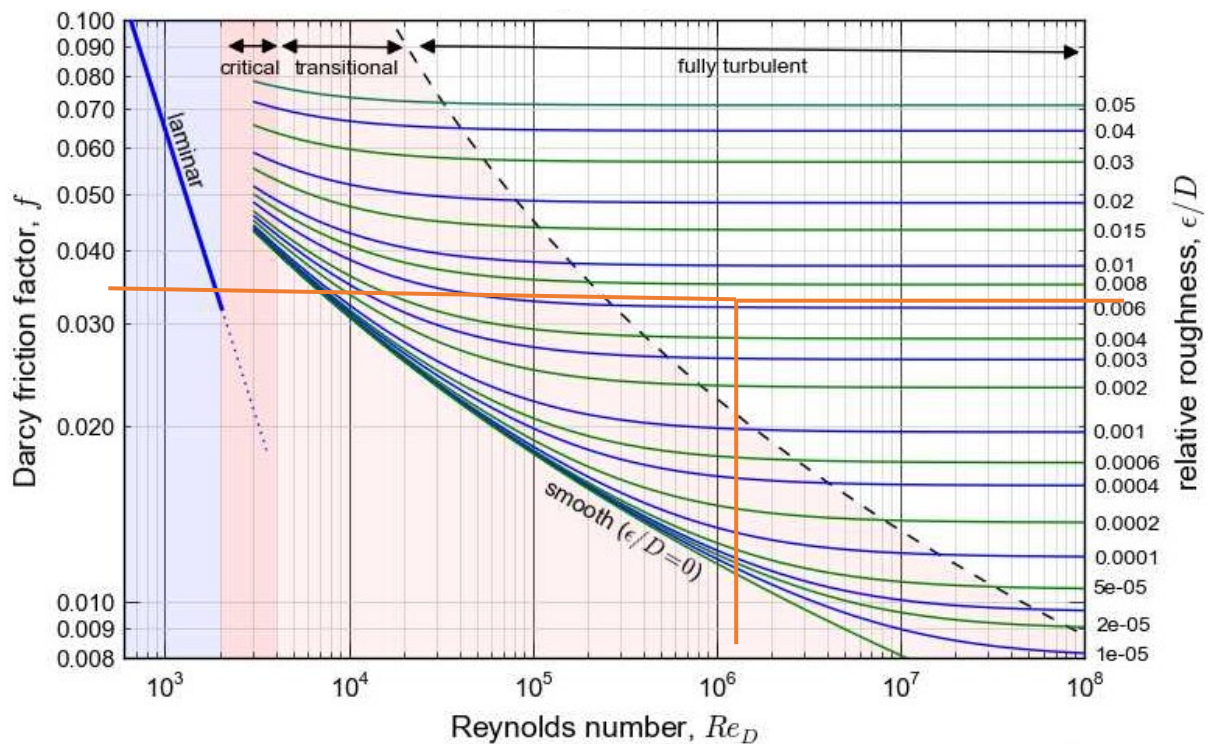


Fig.V.6 Friction factor for fully developed flow in circular pipes(Saad et al., 2024)

Example of reading the Moody diagram in figure V.6. For a Reynolds number equal to 10^6 and a relative roughness $\frac{\epsilon}{D}=0.005$ the coefficient $f=0.03$.

Exercise

Calculate, using the Moody diagram, the linear loss in a commercial steel pipe with a diameter of 700 mm and a length of 400 m, carrying a flow rate of $2.6 \text{ m}^3/\text{s}$.

Generally:

- 💧 The Moody diagram initially covers laminar flows (left side, for low Reynolds numbers). In this case, there is only one straight line, and it is sufficient to read the corresponding value on the graduated axis to the left of the diagram.
- 💧 For intermediate Reynolds numbers, the flow regime is transient, and the same methods as for turbulent flow must be used (without certainty regarding the accuracy of the result).
- 💧 For sufficiently high Reynolds numbers, the flow regime is turbulent. The relative roughness of the pipe's inner wall must first be calculated.
- 💧 A line is chosen from the graph's right side based on the relative roughness (for instance, here for 0.02 or 0.0005).

- Two slightly different turbulent domains are indicated by the central dashed curve.
- In the lower part, the pressure loss coefficient depends on both the Reynolds number and the roughness.
- In the upper part, the pressure loss coefficient becomes independent of the Reynolds number and therefore depends only on the relative roughness (horizontal domain).
- Lastly, we say that we are in a smooth turbulent regime when we are on the lower curve of the turbulent domain. The flow velocity alone, not the roughness, is what causes the turbulence.

V.11. Head loss

Head loss denoted “ H ” is a way of expressing pressure in terms of the height of a fluid column that would produce the same pressure (Madeira, 2020). It consists of:

- The head loss due to viscous effects in the straight pipes, termed the major head loss and denoted “ h_{major} ”.
- and the head loss in the various pipe components, termed the minor head loss and denoted “ h_{minor} ”.

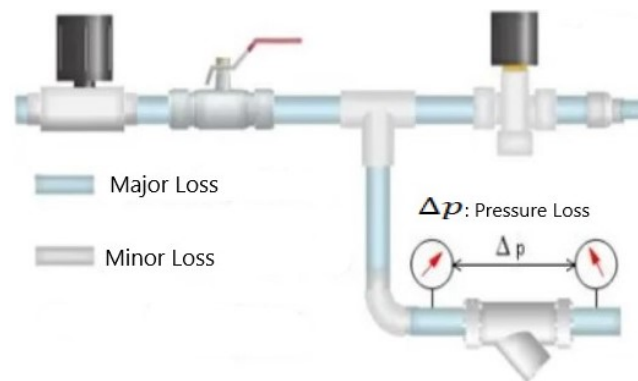


Fig.V.7 Head Loss

That is:

$$H = h_{major} + h_{minor} \quad (V.8)$$

Where:

H head loss or pressure loss in the pipe or duct system

$h_{major-loss}$ is major loss due to friction in the pipe or duct system.

$h_{minor-loss}$ is minor loss due to the components existing in the pipe system.

V.11.1. Major Loss (regular loss)

The **major head loss** are associated with frictional energy loss per length of pipe, are due to friction along the length of a straight pipe, and depends on:

Pipe length, diameter, fluid velocity, viscosity, and roughness.

As shown in figure V. 8 the static pressure (seeing in chapter II) can be related with pressure loss in pipe.

$$\Delta p = \rho g h \quad (\text{II.5})$$

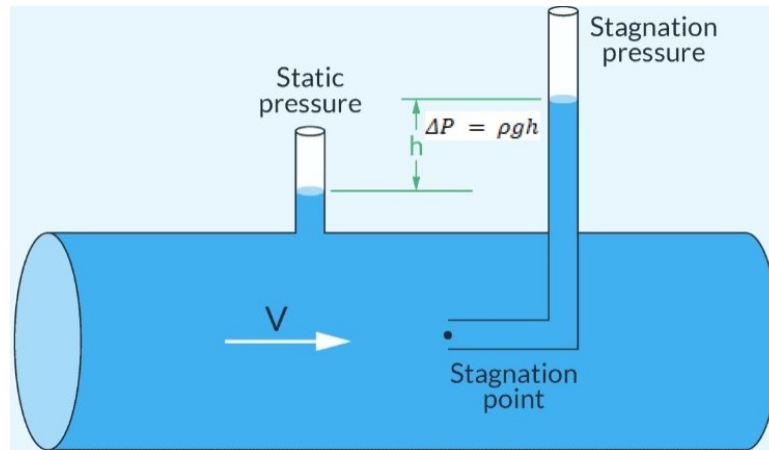


Fig.V.8 Pressure loss Vs static pressure

for a single pipe or duct can be expressed as:

$$\text{Major head} = h_{\text{major}} = \frac{\Delta p}{\rho g} = k \frac{L}{D} \frac{v_{\text{avg}}^2}{2g} [m] \quad (\text{V.9})$$

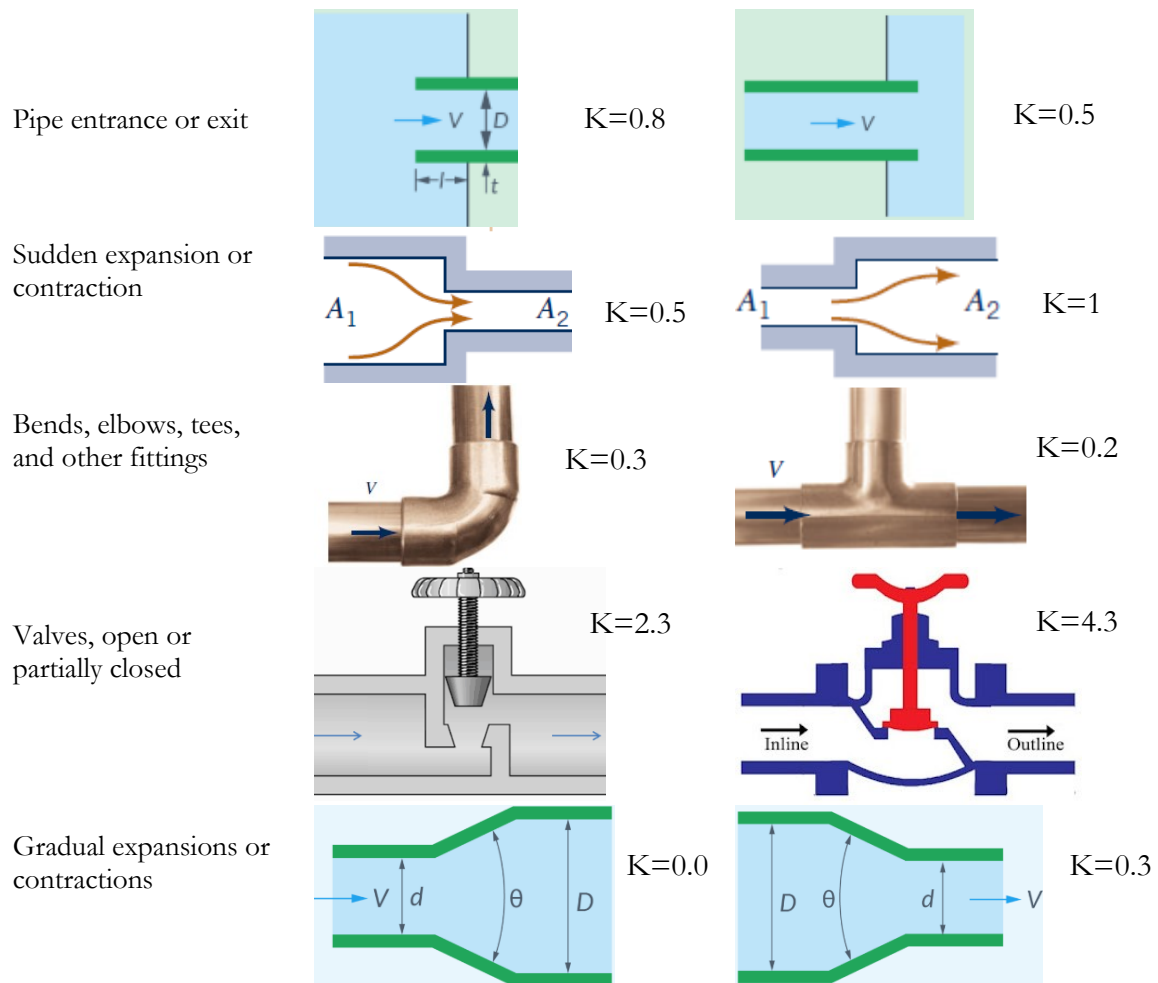
$$\text{Energy loss} = g \cdot h_{\text{major}} = \frac{\Delta p}{\rho} = k \left(\frac{L}{D} \right) \frac{v_{\text{avg}}^2}{2} [J / kg] \quad (\text{V.10})$$

$$\text{regular pressure loss} = \Delta p_{\text{regular}} = \rho \cdot g \cdot h_{\text{major}} = k \left(\frac{L}{D} \right) \cdot \rho \cdot \frac{v_{\text{avg}}^2}{2} [Pa] \quad (\text{V.11})$$

Equation V.9 is valid for duct flows of any cross section and for laminar and turbulent flow. It was proposed by Julius Weisbach, a German professor who in 1850 published the first modern textbook on hydrodynamics(White, 2008).

V.11.2. Minor loss (singular loss)

For any pipe system, in addition to regular (major loss) there are additional so-called minor losses due to fittings, bends, valves, expansions, contractions, or other local disturbances.



In this case pressure loss or min loss is accomplished by changing the geometry of the system i.e., closing or opening the valve alters the flow pattern through the valve.

These minor losses are generally observed in:

- 💧 Restriction devices (a sudden reduction in pipe cross-section)
- 💧 Connecting devices (bends, gradual passage cross-section, etc.)

A singular head loss is expressed as a function of the average velocity V_{avg} before the singularity.

Based on experiments, head losses are proportional to the square of the velocity, and therefore the following form has been adopted:

Singular pressure loss

$$\Delta P [Pa] = k \cdot \rho \cdot \frac{v^2}{2} \quad (IV.12)$$

Minor head in meters of fluid column

$$h_{minor} [m] = k \cdot \frac{v^2}{2g} \quad (V.13)$$

Energy loss

$$g \cdot h_{minor} [J / kg] = k \cdot \rho \cdot \frac{v^2}{2} \quad (V.14)$$

K : The coefficient of singular pressure losses is determined experimentally.

V.12. Bernoulli's theorem applied to a real fluid

Consider a flow between two points ① and ② of a real fluid in a pipe. Assume that there are hydraulic machines between ① and ②.

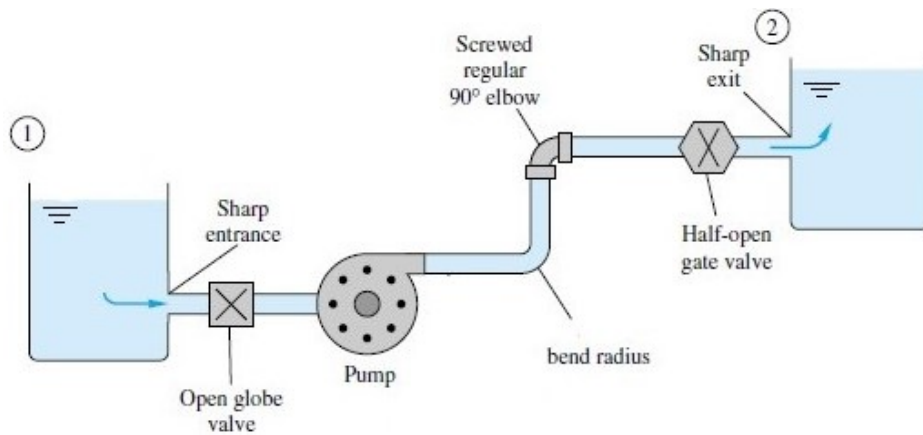


Fig.V.9 Flow in pipe with hydraulic machine and minor losses(White, 2008)

There are three forms to write Bernoulli equation:

V.12.1. Energy form of Bernoulli equation

Bernoulli's equation in the **form of energy** (per unit mass) takes the following expression:

$$\left(\frac{p}{\rho} + \frac{v^2}{2} + gh \right)_1 + \frac{P_{net}}{Q_m} = \left(\frac{p}{\rho} + \frac{v^2}{2} + gh \right)_2 + g \cdot h_{major} + g \cdot h_{minor} \quad (V.12)$$

V.12.2. Pressure form of Bernoulli equation

Bernoulli's equation in the **pressure form** takes the following expression:

$$\left(p + \frac{1}{2} \rho v^2 + \rho gh \right)_1 + \frac{P_{net}}{Q_v} = \left(p + \frac{1}{2} \rho v^2 + \rho gh \right)_2 + \Delta p_{regular} + \Delta p_{singular} \quad (V.13)$$

V.12.3. Head form of Bernoulli equation

In the **head form**, Bernoulli's equation represents the total energy per unit weight of a fluid along a streamline. It is written as:

$$\left(\frac{p}{\rho g} + \frac{v^2}{2g} + h \right)_1 + \frac{P_{net}}{gQ_m} = \left(\frac{p}{\rho g} + \frac{v^2}{2g} + h \right)_2 + H \quad (V.14)$$

Where:

- ◆ $H = h_{major} + h_{minor}$ is the sum of all singular and linear pressure losses between sections (1) and (2).
- ◆ When a hydraulic device is present and exchanges work with the fluid in motion.
 P_{net} is the mechanical power transmitted between the machines and the fluid, potentially positioned between (1) and (2).
 For a Pump: $P_{net} > 0$
 For a Turbine: $P_{net} < 0$
- ◆ Q_m is the mass flow rate.

Bernoulli's equation can be stated in three different forms, depending on whether each term has units of energy, pressure or distance. It can be written as follow:

Energy Form [J/kg]

$$\frac{p}{\rho} + \frac{v^2}{2} + gh = C$$

Pressure Form [Pa]

$$p + \frac{1}{2} \rho v^2 + \rho gh = C$$

Head Form [m]

$$\frac{p}{\rho g} + \frac{v^2}{2g} + h = C$$

V.13. Poiseuille's law

Hagen–Poiseuille formula describes the flow of an incompressible, Newtonian fluid (a fluid with constant viscosity) through a long cylindrical pipe of constant cross-section. This law is used primarily in **fluid dynamics** to predict the volume flow rate of the fluid. (Çengel & Cimbala, 2019)

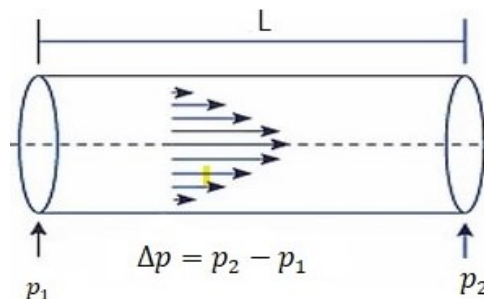


Fig.V.10 Flow rate prediction

The necessary pumping power to overcome the pressure loss is calculated from the pressure loss (or head loss).

for a specified flow rate, the pressure drops and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius (or diameter) of the pipe.

$$P_{pump} = Q_V \Delta p = Q_m \cdot g \cdot h_{major} \quad (V.15)$$

$$Q_V = \frac{\pi}{8} \frac{\Delta p}{\mu} \frac{d^4}{L} \quad (V.16)$$

V.14. Problems Set

Exercise N°1

Heavy fuel oil with a dynamic viscosity $\eta = 0.11 \text{ Pa}\cdot\text{s}$ and a density $d = 0.932$ flows through a pipe of length $L = 1650 \text{ m}$ and diameter $D = 25 \text{ cm}$ at a volumetric flow rate $Q_v = 19.7 \text{ L/s}$.

Required tasks:

- 1) Determine the kinematic viscosity γ of the fuel oil.
- 2) Calculate the flow velocity.
- 3) Deduce the nature of the flow.
- 4) Calculate the energy loss in the pipe.

Solution

- 1) The kinematic viscosity γ of the fuel oil: $\gamma = 118 \cdot 10^{-6} \text{ m}^2/\text{s}$
- 2) The flow velocity: $v = 0.4013 \text{ m/s}$
- 3) The nature of the flow. $N_R = 850.022$; The flow is Laminar
- 4) The energy loss coefficient in the pipe. $k = \frac{64}{N_R} = 0.075$;

So the energy loss equal to $k \left(\frac{L}{D} \right) \frac{v_{avg}^2}{2} = 594 \text{ J/kg}$

Exercise N°2

A coolant circulates in a coiled radiator. The coil comprises the following elements:

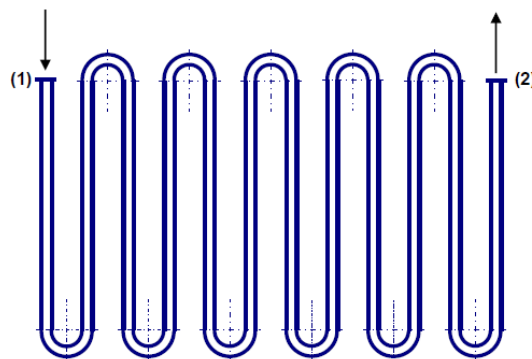
- 💧 12 straight tubes, each with a diameter $d = 10 \text{ mm}$ and a length of 1 m .
- 💧 11 of 180° bends, each with a pressure drop coefficient $k = 0.4$.

The pipe carries a volumetric flow rate $Q_v = 0.25 \text{ L/s}$; The inlet pressure is $P_1 = 3 \text{ bar}$.

The characteristics of the coolant are given: $\eta = 10^{-3} \text{ Pa}\cdot\text{s}$; $\rho = 1000 \text{ kg/m}^3$.

Required work :

- 1) Specify the nature of the flow.
- 2) Calculate the head losses:
- 3) Determine P_2



Solution

1) The nature of the flow: $v = 3.183 \text{ m/s}$;

$N_R = 850.022$; The flow is turbulent.

2) The head losses: $H = h_{major} + h_{minor}$

$$h_{major} = \sum(f) \cdot \frac{L}{D} \frac{V_{avg}^2}{2g} ; f = 0.316 N_R^{-0.25} = 0.02366 ; h_{major} = 0.02366 \times \frac{12 \times (1)}{10^{-2}} \frac{(3.183)^2}{2 \times 9.81} = 14.68 \text{ m}$$

$$\Delta p_{maj} = \rho g h_{maj} \approx 1000 \cdot 9.81 \cdot 14.68 \approx 1.44 \times 10^5 \text{ Pa } (\approx 144 \text{ kPa}).$$

$$h_{minor} = \sum(k) \cdot \frac{v^2}{2g} = 11 \times (0.4) \times \frac{(3.183)^2}{2 \times 9.81} = 2.272 \text{ m}$$

$$\Delta p_{min} = \rho g h_{min} \approx 1000 \cdot 9.81 \cdot 2.27 \approx 2.23 \times 10^4 \text{ Pa } (\approx 22.3 \text{ kPa}).$$

3) The out let pressure: $\left(\frac{p}{\rho g} + \frac{v^2}{2g} + z \right)_1 = \left(\frac{p}{\rho g} + \frac{v^2}{2g} + z \right)_2 + h_{major} + h_{minor}$; knowing that: $v_1 = v_2$ and

$$z_1 = z_2$$

$$\text{so } P_2 = P_1 + \rho \cdot g \cdot (h_{maj} + h_{min}) = 1.342 \text{ Bar}$$

Exercise N°3

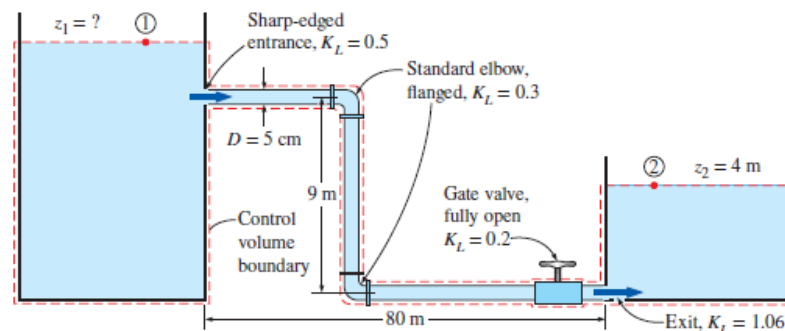
Water at 10°C flows from a large reservoir to a smaller one through a 5cm-diameter cast iron piping system, as shown in the Figure.

The piping system involves 89 m of piping length, a sharp-edged entrance ($K = 0.5$), 2 elbows ($K = 0.3$ each), a fully open gate valve ($K = 0.2$), and a submerged exit ($K = 1.06$).

Determine the elevation z_1 for a flow rate of 6 L/s .

Properties The density and dynamic viscosity of water at 10°C are:

$\rho = 999.7 \text{ kg/m}^3$ and $\eta = 1.307 \times 10^{-3} \text{ kg/m.s}$. The roughness of cast iron pipe is $\varepsilon = 0.00026 \text{ m}$



Solution

Bernoulli between free-surface points ① and ②:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_{major} + h_{minor}$$

Knowing that : $v_1 \approx v_2$; $p_1 \approx p_2$

$$z_1 = z_2 + h_{major} + h_{minor} = z_2 + \left(f \frac{L}{D} + \sum k \right) \frac{v^2}{2g}$$

So

The velocity number of the piping system is equal to:

$$V = \frac{Q_v}{A} = \frac{0.006}{1.9634954 \times 10^{-3}} \approx 3.05577 \text{ m/s.}$$

$$\text{The Cross-sectional area: } A = \frac{\pi D^2}{4} = \frac{\pi (0.05)^2}{4} = 1.9634954 \times 10^{-3} \text{ m}^2$$

$$\text{The Reynolds number: } N_R = \frac{\rho u \cdot D_{hyd}}{\eta} = \frac{3.05577 \times 0.05}{1.30739 \times 10^{-6}} \approx 1.16865 \times 10^5. \text{ So the flow is turbulent.}$$

$$\text{the Major loss coefficient: } f = 0.79 \sqrt{\frac{\varepsilon}{D}} = 0.03152.$$

$$\text{The Major (regular) head loss: } h_{maj} = f \frac{L}{D} \frac{v^2}{2g} \approx 0.03152 \cdot \frac{89}{0.05} \cdot \frac{(3.05577)^2}{2 \cdot 9.81} \approx 26.701 \text{ m.}$$

$$\text{The Minor (singular) head loss: } h_{minor} = k \cdot \frac{v^2}{2g}$$

$$\text{First sum the } K \text{ values: } K_{Tot} = K_e + 2K_{elbow} + K_v + K_{exit} = 0.5 + 2(0.3) + 0.2 + 1.06 = 2.36.$$

$$\text{Then: } h_{minor} = k \cdot \frac{v^2}{2g} = 2.36 \cdot \frac{(3.05577)^2}{2 \cdot 9.81} \approx 1.12320 \text{ m.}$$

$$\text{Total loss: } h_{Tot} = h_{maj} + h_{min} \approx 26.701 + 1.1232 = 27.8246 \text{ m.}$$

Corresponding pressure drop

$$\Delta p_{Total} = \rho g h_{Tot} \approx 999.7 \times 9.81 \times 27.8246 \approx 272,900 \text{ Pa } (\approx 272.9 \text{ kPa}).$$

Finally, required upstream free-surface elevation

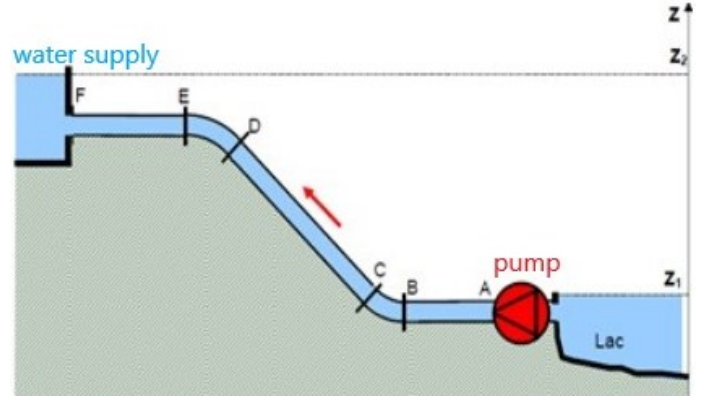
$$z_1 = z_2 + h_{Tot} = 4.00 + 27.8246 \approx 31.82 \text{ m.}$$

Exercise N°4

A pump with a volumetric flow rate $q_v = 2 \text{ L/s}$ and an efficiency $\eta = 70\%$ pumps water from a lake to a reservoir located on a hill, as shown in the figure.

The water is conveyed through a pipe with a diameter $d = 130 \text{ mm}$, consisting of three straight sections:

- AB, length $L_1 = 10 \text{ m}$,
- CD, length $L_2 = 12 \text{ m}$,
- EF, length $L_3 = 8 \text{ m}$,
- two 45° bends: BC and DE, each with a head loss coefficient $K_s = 0.33$.
- $z_1 \approx 0$ and $z_2 \approx 10 \text{ m}$.



- 1) Specify the nature of the flow.
- 2) Determine the net power P_{net} of the pump
- 3) Deduce the power P_a absorbed by the pump

Solution

- 1) Nature of the flow

☞ Velocity: $V = \frac{Q_v}{A} = \frac{0.002}{0.01327} \approx 0.1507 \text{ m/s}$

☞ Reynolds number: $N_R = \frac{VD}{\nu} \approx \frac{0.1507 \times 0.13}{1 \times 10^{-6}} \approx 1.96 \times 10^4$.

Since $N_R \gg 2300$ the flow is **turbulent**

- 2) Friction factor: No roughness given $\rightarrow$ use Blasius:

$$f \approx 0.3164 N_R^{-1/4} \approx 0.3164 \times (1.96 \times 10^4)^{-1/4} \approx 0.0267.$$

- 3) Head losses:

Major Loss (Darcy–Weisbach):

Total straight length $L = L_1 + L_2 + L_3 = 10 + 12 + 8 = 30 \text{ m}$

$$h_{maj} = f \frac{L}{D} \frac{V^2}{2g} = 0.0267 \cdot \frac{30}{0.13} \cdot \frac{(0.1507)^2}{2 \cdot 9.81} \approx 7.12 \times 10^{-3} \text{ m}.$$

Minor (singular):

Bends: two 45° bends, $K_{Tot} = 2 \times 0.33 = 0.66$.

$$h_{min} = K_{Tot} \frac{V^2}{2g} = 0.66 \cdot \frac{(0.1507)^2}{2 \cdot 9.81} \approx 7.65 \times 10^{-4} \text{ m}.$$

Total hydraulic losses: $h_{loss} = h_{maj} + h_{min} \approx 0.00712 + 0.00076 \approx 0.00788 \text{ m}.$

- 4) Pump net hydraulic power P_{net} :

Equation Bernoulli:
$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + \frac{P_{net}}{gQ_m} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_{major} + h_{minor}$$

Or $v_1 = v_2$ and $p_1 = p_2 = p_{atm}$.

Hydraulic (net) power delivered to the fluid:

$$P_{net} = \rho g Q_v (h_{maj} + h_{min}) = 1000 \cdot 9.81 \cdot 0.002 \cdot 10.0079 \approx 196.2 \text{ W.}$$

Round to sensible precision: $P_{net} \approx 196 \text{ W}$.

5) Power absorbed by the pump (shaft/electrical) P_a

Given pump efficiency $\eta = 70\% = 0.70$ Power required at pump shaft (input) is

$$P_a = \frac{P_{net}}{\eta} = \frac{196.2}{0.70} \approx 280.3 \text{ W.}$$

VI. APPENDICES

VI.1. Appendices

VI.1.1. Mathematical reminders

VI.1.1.1. Vectors : (Marc, 2023)

Let $\vec{A}$ and $\vec{B}$ be two vectors in $\mathbb{R}^2$: The scalar product is equal to:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = \sum_{i=1}^2 A_i B_i \quad (\text{VI.1})$$

Let $\vec{A}$ and $\vec{B}$ be two vectors in oriented space. We define their vectorial-product $\vec{A} \wedge \vec{B}$ by:

$$\vec{C} = \vec{A} \wedge \vec{B} = A_x B_y - A_y B_x \quad (\text{VI.2})$$

The tensor product, which is a matrix:

$$\overline{\overline{M}} = \vec{A} \otimes \vec{B} = \begin{bmatrix} A_x B_x & A_x B_y \\ A_y B_x & A_y B_y \end{bmatrix} \quad (\text{VI.3})$$

VI.1.1.1.2. Scalar function $f(x,y,t)$ of 2 spatial variables x,y and time t

Example: $f(x,y,t) = x^2 y e^{-t}$

Its differential is a scalar: $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial t} dt = 2xye^{-t} dx + x^2 e^{-t} dy - x^2 y e^{-t} dt$

Its gradient is a vector of components: $\left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$ So $\overrightarrow{\text{grad}} f = \frac{\partial f}{\partial x} \vec{e}_x + \frac{\partial f}{\partial y} \vec{e}_y = \begin{bmatrix} 2xye^{-t} \\ x^2 e^{-t} \end{bmatrix}$

VI.1.1.2. 2D Vector Function

If $\vec{U}(x,y,t)$ is a vector field in $\mathbb{R}^2$ space and time $u(x,y,t)$ and $v(x,y,t)$ are its components in 2D direction in X and Y direction.

◆ Then the divergence of $\vec{U}(x,y,t)$ is defined by: $\text{div} \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

It can be written symbolically $\text{div} \vec{U} = \vec{\nabla} \cdot \vec{U}$

Example: $\vec{U}(x,y,t) = \begin{bmatrix} 2xye^{-t} \\ (x^2 + y^2)e^t \end{bmatrix}$

Its divergence is a scalar: $\text{div} \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2ye^{-t} + 2ye^t$

Note: Divergence reflects the compression or expansion of the field

- ◆ Its **curl** is a vector perpendicular to XY plane and its amplitude is defined as:

$$\text{curl} \vec{U} = \vec{\nabla} \wedge \vec{U} = \begin{vmatrix} \vec{i} & \vec{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ u & v \end{vmatrix}$$

$$\text{So } \overrightarrow{\text{Curl}U} = \left\{ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right\} \vec{k}$$

Note: The **curl** represents the solid rotation of the field

VI.1.1.3. Incompressible Euler and Navier-Stokes equation (Boyer & Fabrie, 2005)

A flow is said to be incompressible if one of the following equivalent properties is satisfied:

The volume of any fluid element is constant over time;

The velocity field v has zero divergence: $\text{div } v \in 0$;

The density is constant along the trajectories.

$$\begin{cases} \text{div} V = 0 \\ \partial_t V + (v \cdot \nabla) v + \nabla p - \nu \Delta v = f \end{cases}$$

Where " ν " is called kinematic viscosity, and the "pressure" p (which is no longer a function of density) is understood as a Lagrange multiplier associated with the incompressibility stress $\text{div } v \in 0$.

VI.1.1.4. Euler equations for homogeneous incompressible

In the case of a non-viscous, incompressible, homogeneous fluid (sometimes called a perfect fluid), we obtain the following system:

The vorticity within the flow is defined as follows: $\omega = \text{curl}(v)$

$$\text{In 2D: } \text{curl}(v) = \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2} \in \mathbb{R}$$

$$\text{In 3D: } \text{curl}(v) = \begin{bmatrix} \frac{\partial v_3}{\partial x_2} - \frac{\partial v_2}{\partial x_3} \\ \frac{\partial v_1}{\partial x_3} - \frac{\partial v_3}{\partial x_1} \\ \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2} \end{bmatrix} \in \mathbb{R}^3$$

Note: In French, the **curl** operator is called **rot**.

VI.1.2. Fluid propreity tables

Table VI.1 : Common engineering fluids at 20°C and 1 atm:

Fluid	Density, ρ (kg/m ³)	Dynamic Viscosity, μ (N. s/m ²)	Kinematic Viscosity, ν (m ² /s)
Water	998	1.01E-03	1.01E-06
Air	1.21	1.84E-05	1.54E-05
Engine Oil	888.1	0.8374	9.429×10^{-4}
Glycerin	1264	1.519	1.201×10^{-3}

Table VI.2– Thermophysical properties of water (Yunus, 2010)

Temperature (°C)	Density ρ (kg·m ⁻³)	Spécific Heat c_p (kJ·kg ⁻¹ ·K ⁻¹)	thermal Conductivity k (W·m ⁻¹ ·K ⁻¹)	Dynamic Viscosity μ (Pa·s)	kinematic Viscosity ν (m ² ·s ⁻¹) $\times 10^{-6}$
0	999.8	4.217	0.561	1.79×10^{-3}	1.79
10	999.7	4.192	0.580	1.31×10^{-3}	1.31
20	998.2	4.182	0.598	1.00×10^{-3}	1.00
30	995.7	4.178	0.615	0.80×10^{-3}	0.80
40	992.2	4.178	0.630	0.65×10^{-3}	0.65
60	983.2	4.183	0.653	0.47×10^{-3}	0.48
80	971.8	4.195	0.670	0.36×10^{-3}	0.37
100	958.4	4.216	0.679	0.28×10^{-3}	0.29

Table VI.3 : Physical Properties of Common Liquids at 20°C (Fox et al., 2020)

Liquid	Isentropic Bulk Modulusa (GN/m ²)	Specific Gravity (—)
Water	2.24	0.998
Seawater	2.42	1.025
Glycerin	4.59	1.26
Castor oil	2.11	0.969
Benzene	1.48	0.879
Carbon tetrachloride	1.36	1.595

VI.1.3. Physical Constants

Physical Constants Used in Incompressible Fluid Dynamics:

1. Gravitational Acceleration

Symbol: g ; Value: $g = 9.81$

Role: Appears in hydrostatic pressure, Bernoulli equation, and free-surface flows.

2. Reference Atmospheric Pressure

Symbol: p_{atm} ; Value: $p_{\text{atm}} = 101\,325$ Pa

Role: Used as reference pressure in open-flow problems and gauge pressure calculations.

3. Fluid Density (Constant for Incompressible Flow)

Symbol: ρ ; Typical values: Water (20 °C): 998

Air (treated as incompressible at low speed): 1.2

Role: Central parameter in momentum, continuity, and energy equations.

4. Dynamic Viscosity

Symbol: μ ;

Typical values: Water (20 °C): 1.0×10^{-3} Pa·s

Role: Governs shear stresses and viscous dissipation.

5. Kinematic Viscosity

Symbol: ν

Definition: $\nu = \frac{\mu}{\rho}$

Typical value (water, 20 °C): $\nu \approx 1.0 \times 10^{-6}$ m²/s

Role: Used in Reynolds number and viscous flow analysis.

6. Surface Tension (when free surfaces are present)

Symbol: σ

Typical value (water–air, 20 °C): $\sigma = 0.0728$ N·m⁻²

Role: Important in capillary flows, droplets, and microfluidics.

7. Thermal Constants (when coupling with heat transfer)

Constant	Symbol	Typical value (water, 20 °C)
Specific heat	c_p	4182
Thermal conductivity	k	0.6

Bibliography

Bibliography



- Al Makky, A. (2007). *Burning Rate Characteristics of Turbulent Rich Premixed Flames* [PhD Thesis, Cardiff University].
https://cfd2012.com/uploads/3/5/1/0/35107563/master_thesis_2007_ahmed_al_makky.pdf
- Anderson, J. D. (1992). Governing Equations of Fluid Dynamics. In J. F. Wendt (Ed.), *Computational Fluid Dynamics* (pp. 15–51). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-662-11350-9_2
- Baylar, A., & Ozkan, F. (2006). Applications of Venturi Principle to Water Aeration Systems. *Environmental Fluid Mechanics*, 6(4), 341–357. <https://doi.org/10.1007/s10652-005-5664-9>
- Beauvais, R. (1994). Laser-Doppler Velocimetry (LDV). In F. Mayinger (Ed.), *Optical Measurements* (pp. 179–193). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-662-02967-1_10
- Bellakhal, G. (2015). *MECANIQUE DES FLUIDES*.
https://utm.rnu.tn/visirech/Fr/utm/DOWNLOAD_1449248695.pdf
- BERIACHE, D. M. (2019). *MECANIQUE DES FLUIDES II*. https://www.univ-chlef.dz/ft/wp-content/uploads/2020/04/Polycopi%C3%A9_MDF2.pdf
- Boyer, F., & Fabrie, P. (2005). *Eléments d'analyse pour l'étude de quelques modèles d'écoulements de fluides visqueux incompressibles* (Vol. 52). Springer Science & Business Media.
<https://books.google.fr/books?hl=en&lr=&id=Q7T4xRcsEYYC&oi=fnd&pg=PA1&dq=%C3%A9ments+d%E2%80%99analyse+math%C3%A9matique+des+fluides+incompressibles+&ots=K1f2RmerpK&sig=35QFvplMzMqIaIikoH8TbmAXFqk>
- Cengel, Y. A., & Cimbala, J. M. (2017). *Fluid mechanics: Fundamentals and applications* (p. 956). New York: McGraw-Hill Education.

Çengel, Y. A., & Cimbala, J. M. (2019). *Fluid Mechanics: Fundamentals and Applications, 4e in SI Units*.

McGraw-Hill Education.

Chanson, H. (2009). *Applied hydrodynamics: An introduction to ideal and real fluid flows*. CRC press.

https://books.google.com/books?hl=fr&lr=&id=bLIGhhgqaUgC&oi=fnd&pg=PP1&dq=Ideal+fluid+vs+Real+fluid&ots=L7WT'_sAcj&sig=9IppyETjqWpdi2XjHSYQrDPSpNY

Cüneyt, S. (2012). *ME 305 Fluid Mechanics I Part 2 Fluid statics*. SCRIBD.

<https://www.scribd.com/document/94833617/ME-305-Part-2-Fluid-Statics>

Dourmashkin, P. A. (2014). *Classical mechanics: Mit 8.01 course notes*. Wiley Custom Learning Solutions.

Elger, D. F., LeBret, B. A., Crowe, C. T., & Roberson, J. A. (2020). *Engineering fluid mechanics*. John Wiley & Sons.

https://books.google.com/books?hl=fr&lr=&id=ljP6DwAAQBAJ&oi=fnd&pg=PP7&dq=To+determine+the+density+at+a+point,+we+must+select+a+small+volume,+%F0%9D%9B%BF%F0%9D%91%89,+surrounding+the+point+and+then+determine+the+ratio+%F0%9D%9B%BF%F0%9D%91%89.%F0%9D%91%89.&ots=LNNIheJ3Au&sig=VMSh8USe0nqa0bHtGeo-_hupY3Q

Fannelöp, T. K. (2013). *Fluid mechanics for industrial safety and environmental protection* (Vol. 3). Newnes.

https://books.google.fr/books?hl=en&lr=&id=GE5NBjmO74AC&oi=fnd&pg=PP1&dq=Example:+In+industrial+piping+for+chemical+plants,+knowing+the+velocity+profile+prevents+erosion+of+the+pipe+walls+and+ensures+safe+transport+of+reactive+fluids.&ots=juDs5X3hVX&sig=Ho9mKJbpLyY_w51RJTvHJ3O19Do

Fox, R. W., McDonald, A. T., & Mitchell, J. W. (2020). *Fox and McDonald's introduction to fluid mechanics*.

John Wiley & Sons.

Genick Bar–Meir, P. D. (2013). Basics of Fluid Mechanics. *Version (0.3. 0.3 December 5, 2010)*.

<http://cdn.worldcolleges.info/sites/default/files/fluidMechanics.pdf>

Graf, E. H. (2004). Just what did Archimedes say about buoyancy? *The Physics Teacher*, 42(5), 296–299.

- Grant, H. L., Moilliet, A., & Stewart, R. W. (1959). A spectrum of turbulence at very high Reynolds number. *Nature*, 184(4689), 808–810.
- Halliday, D., Resnick, R., & Krane, K. S. (2010). *Physics, Volume 2*. John Wiley & Sons.
<https://books.google.com/books?hl=fr&lr=&id=hAZ-DwAAQBAJ&oi=fnd&pg=PA567&dq=University+Physics+Volume+1&ots=Vxw'TDO3U-u&sig=dvOGlx46KaFywaguKRbyKQ9zXm4>
- Hamouda, R. B. (2009). *Notions de mécanique des fluides: Cours et exercices corrigés*. UVT.
- Higham, E. H., & Paros, J. M. (2003). Measurement of pressure. In *Instrumentation Reference Book* (pp. 123–143). Elsevier. <https://www.sciencedirect.com/science/article/pii/B9780750671231500132>
- Jiang, G. U. O., Qikai, L. I., Pu, Q. I. N., Ankang, Y., Mingyang, L. U., Xiaolong, K. E., & Benny, C. F. (2025). Internal surface finishing and roughness measurement: A critical review. *Chinese Journal of Aeronautics*, 38(8), 103303.
- Kiselev, S. P., Vorozhtsov, E. V., & Fomin, V. M. (1999). Ideal Fluid. In S. P. Kiselev, E. V. Vorozhtsov, & V. M. Fomin, *Foundations of Fluid Mechanics with Applications* (pp. 187–266). Birkhäuser Boston.
https://doi.org/10.1007/978-1-4612-1572-1_4
- Madeira, A. A. (2020). Major and minor head losses in a hydraulic flow circuit: Experimental measurements and a Moody's diagram application. *Eclética Química*, 45(3), 47–56.
- Marc, B. (2023). *Cours de mécanique des fluides approfondie*. https://perso.univ-lyon1.fr/marc.buffat/COURS/BOOK_MECAFLU
- MARCHESSE, Y. (2019). Mécanique des fluides. *ECAM-LYON, Lyon, France*. https://elearning.centre-mila.dz/a2025/pluginfile.php/25858/mod_resource/content/1/A%C3%A9rodynamique%20page%20178.pdf
- McDonough, J. M. (2009). *Lectures in elementary fluid dynamics: Physics, mathematics and applications*.
https://uknowledge.uky.edu/cgi/viewcontent.cgi?article=1000&context=me_textbooks

- Monchwe, T. B. (2023). Bernoulli's principle and the Venturi effect. *Southern African Journal of Anaesthesia and Analgesia*, 42–44.
- Munson, B. R., Young, D. F., & Okiishi, T. H. (2013). Fundamentals of fluid mechanics. *Oceanographic Literature Review*, 10(42), 831.
- Nakayama, Y. (2018). *Introduction to fluid mechanics*. Butterworth-Heinemann.
- Nawale, H. (2021). *CASE STUDY ON SIMPLE CARBURETTOR*.
https://www.academia.edu/download/68962383/IRJET_V8I3585.pdf
- Oertel, H. (2004). *Prandtl's essentials of fluid mechanics*. Springer.
https://link.springer.com/content/pdf/10.1007/0-387-21803-3_4.pdf
- Ogunsona, E., Ojogbo, E., & Mekonnen, T. (2018). Advanced material applications of starch and its derivatives. *European Polymer Journal*, 108, 570–581.
- Panton, R. L. (2024). *Incompressible flow*. John Wiley & Sons.
<https://books.google.com/books?hl=fr&lr=&id=T77sEAAAQBAJ&oi=fnd&pg=PR12&dq=Incompressible+Fluid:+Dynamics+and+Fundamental+Concepts&ots=ucX7MmdBo2&sig=mPvwEVBtFr8o6Gagkw9tcBGJJAg>
- Papanastasiou, T., Georgiou, G., & Alexandrou, A. N. (2021a). *Viscous fluid flow*. CRC press.
<https://books.google.com/books?hl=fr&lr=&id=fem3OFFC6RQC&oi=fnd&pg=PR11&dq=+VISCOUS+FLUID+FLOW+Tasos+C.+Papanastasiou+Georgios+C.+Georgiou&ots=6cfUGleYmK&sig=INWXPmFldozb6Uy7kYhGagZ-Kqc>
- Papanastasiou, T., Georgiou, G., & Alexandrou, A. N. (2021b). *Viscous fluid flow*. CRC press.
<https://books.google.com/books?hl=fr&lr=&id=90coEAAAQBAJ&oi=fnd&pg=PP1&dq=definition+of+real+viscous+flow+&ots=yddPA2t8Dk&sig=H8YUEq8r1zJchjdHtDezafCN8JU>
- Pascal, B. (2020). *CIRA- MESURE DU DEBIT*. <https://docplayer.fr/20695892-Cira-1-les-debits-1-pascal-bigot-la-mesure-du-debit-dans-ce-chapitre-nous-abordons-les-methodes-de-mesure-des-vitesses-et-des-debits.html>

- Paul, A., Laurila, T., Vuorinen, V., & Divinski, S. V. (2014). Fick's Laws of Diffusion. In A. Paul, T. Laurila, V. Vuorinen, & S. V. Divinski, *Thermodynamics, Diffusion and the Kirkendall Effect in Solids* (pp. 115–139). Springer International Publishing. https://doi.org/10.1007/978-3-319-07461-0_3
- Poling, B. E., Prausnitz, J. M., John Paul, O., & Reid, R. C. (2001). *The properties of gases and liquids* (Vol. 5). Mcgraw-hill New York. https://www.academia.edu/download/62015682/Prausnitz_-_Properties_of_Gases_and_Liquids20200206-55970-35et18.pdf
- Potter, M. C., & Wiggert, D. C. (2021). *Schaum's outline of fluid mechanics*. McGraw-Hill Education.
- Qin, R., & Duan, C. (2017). The principle and applications of Bernoulli equation. *Journal of Physics: Conference Series*, 916(1), 012038. <https://iopscience.iop.org/article/10.1088/1742-6596/916/1/012038/meta>
- Saad, J. C. C., Rojo, D. P. G., & Gaia, C. D. C. (2024). FRICTION FACTOR DETERMINATION IN DIFFERENT REGIONS OF THE MOODY CHART. *IRRIGA*, 29, 68–77.
- Sachin Thorat. (n.d.). *Types of Flows in a Pipe / Fluid Mechanics* [Letter]. Retrieved <https://learnmech.com/types-of-flows-in-pipe-fluid-mechanics/>
- Smits, A. J. (2012). *Flow visualization: Techniques and examples*. World Scientific. <https://books.google.fr/books?hl=en&lr=&id=WDW6CgAAQBAJ&oi=fnd&pg=PR5&dq=Flow+Visualization+%26+Measurement&ots=UUUv3BMhTR&sig=OOylkzE9sObs7ZxwelUKxVbvcvo>
- Song, H. (2018). *Engineering Fluid Mechanics*. Springer Singapore. <https://doi.org/10.1007/978-981-13-0173-5>
- Sultanian, B. K. (2021). Bernoulli Equation: Mechanical Energy Equation. In *Fluid Mechanics and Turbomachinery* (pp. 49–66). CRC Press. <https://www.taylorfrancis.com/chapters/edit/10.1201/9781003053996-3/bernoulli-equation-bijay-sultanian>
- Treiber, M., & Kesting, A. (2025). Continuity Equation. In M. Treiber & A. Kesting, *Traffic Flow Dynamics* (pp. 109–125). Springer Nature Switzerland. https://doi.org/10.1007/978-3-031-93922-8_8

Vogel, S. (1996). *Life in moving fluids: The physical biology of flow*. Princeton university press.

<https://books.google.com/books?hl=fr&lr=&id=XBqncfXFfOIC&oi=fnd&pg=PR9&dq=Stagnation+points+on+bodies+in+flowing+fluids+books&ots=RpXuRkZwmN&sig=DWdcLMvGgU3se4ewE5zWgrnt36A>

Wang, Z. B., & Zheng, Y. G. (2021). Critical flow velocity phenomenon in erosion-corrosion of pipelines: Determination methods, mechanisms and applications. *Journal of Pipeline Science and Engineering*, 1(1), 63–73.

Westerhof, N., Stergiopulos, N., Noble, M. I. M., & Westerhof, B. E. (2019). Viscosity. In N. Westerhof, N. Stergiopulos, M. I. M. Noble, & B. E. Westerhof, *Snapshots of Hemodynamics* (pp. 3–8). Springer International Publishing. https://doi.org/10.1007/978-3-319-91932-4_1

White, F. M. (2008). *Fluid mechanics*. The McGraw Hill Companies,.
<http://103.62.146.201:8081/xmlui/handle/1/9343>

Yunus, A. C. (2010). *Fluid Mechanics: Fundamentals And Applications (Si Units)*. Tata McGraw Hill Education Private Limited.

Zhang, J. (2025). Aerodynamic Characteristics of Different Types of Vehicles. *2025 2nd International Conference on Electrical Engineering and Intelligent Control (EEIC 2025)*, 858–865. <https://www.atlantispress.com/proceedings/eeic-25/126016694>

Electronic web site

<https://www.piping-designer.com/properties/classical-mechanics/469-mass-diffusivity>

<https://www.test-and-measurement-world.com/articles/general/ideal-fluid-vs-real-fluid>

<https://www.pearson.com/channels/physics/learn/patrick/fluid-mechanics/ideal-real-fluids>

[Fluid Dynamics - University Physics Volume 1 | OpenStax](#)

[Bernoulli's Principle & Bernoulli Equation - Definition, Derivation, Principle of Continuity, Applications, Examples and FAQs \(byjus.com\)](#)

<https://www.nuclear-power.com/nuclear-engineering/fluid-dynamics/bernoullis-equation-bernoullis-principle/>

<http://ecoursesonline.iasri.res.in/mod/page/view.php?id=1080>

<https://engineerexcel.com/incompressible-fluid/>

<https://pediaa.com/difference-between-compressible-and-incompressible-fluids/>