Ministry of Higher Education and Scientific Research
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L2 Physics

THERMODYNAMIC Exercises and solutions

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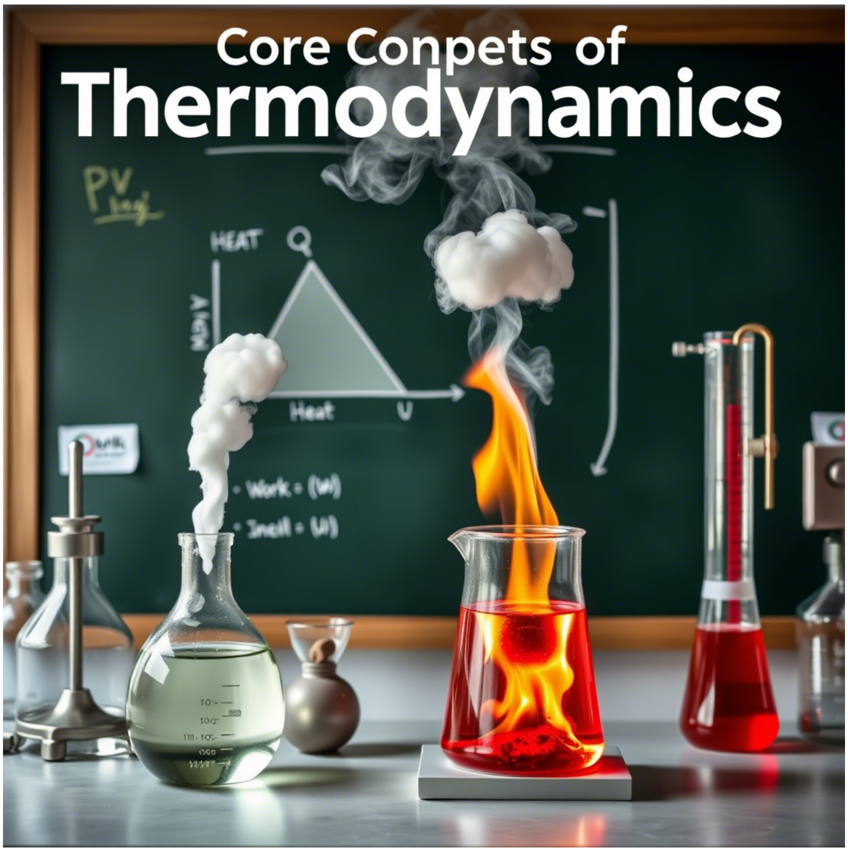
Core Conpets of
Thermodynamics
PV
HEAT Q
Heat
V
• Work = (W)
• Inell = (U)

Contents

Introduction

This collection of exercises and their corresponding solutions is designed as a supplementary resource for the **Thermodynamics** module, primarily intended for **second-year undergraduate students** pursuing a **Licence in Physics**. The aim of this compilation is to reinforce theoretical concepts through practical problem-solving, bridging the gap between classroom instruction and independent study.

Thermodynamics, as a fundamental branch of physics, governs the principles of energy, heat, and work in physical systems. Mastery of this subject is essential for students, as it forms the basis for advanced studies in statistical mechanics, fluid dynamics, and engineering applications. However, many learners encounter challenges in applying abstract thermodynamic laws to concrete problems. This workbook seeks to address that difficulty by providing a structured set of exercises ranging from basic to moderately advanced levels, each accompanied by a detailed solution.

The exercises are carefully selected to align with standard university curricula, covering key topics such as:

- The First and Second Laws of Thermodynamics,
- Thermodynamic potentials and state functions,
- Heat engines, refrigerators, and efficiency calculations,
- Phase transitions and equations of state,
- Introduction to statistical thermodynamics (where applicable).

Target Audience: While this material is broadly useful for university students in physical sciences, it is **primarily tailored** for second-year Licence students in Physics, who are encountering thermodynamics in depth for the first time. Educators may also find this compilation helpful as a teaching aid for tutorials and practice sessions.

Each problem is solved methodically, emphasizing logical reasoning and step-by-step derivations to cultivate a strong problem-solving intuition. Additionally, special attention is given to common pitfalls and misconceptions, ensuring a robust conceptual foundation.

We hope this exercise collection serves as a valuable tool in your academic journey, helping you develop both confidence and competence in thermodynamics.


Thermodynamics
Thermodynamics

The classification of exercices

The classification that allows for quick navigation to the exercises corresponding to each thermodynamics topic.

Equations of State and Ideal Gases

Exercise 1: Equation of state of ideal gases, unit of R, molar volume

Exercise 2: Equation of state in specific volume, constant $r = R/M$

Exercise 3: Relative volume variation (ideal gas law)

Exercise 4: Van der Waals equation, units of a and b

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Mathematical Relations and Differentials

Exercise 5: Cyclic relation $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$

Exercise 6: Exact differential, integrating factor

Exercise 7: Exact or inexact differentials, path-dependent calculation

Exercise 8: Reech formulas and relations between partial derivatives

Thermoelastic Coefficients

Exercise 6: Coefficients α, β, χ for an ideal gas

Exercise 7: Expansion and compressibility coefficients from a state equation

Exercise 8: Pressure calculation to maintain constant volume

Exercise 9: Relations between C_P, C_V, l, h and second derivatives

First Law of Thermodynamics

Exercise 11: Expression of δQ in variables $(T, V), (T, P), (P, V)$

Exercise 12: δQ is not an exact differential

Exercise 13: Calculation of ΔU in different situations

Exercise 14: Exactness of dU and dS, ideal gas case

Exercise 15: Work exchanged in a Van der Waals transformation

Exercise 16: Work in a cycle of transformations

Exercise 17: Work in reversible and irreversible isothermal transformations

Exercise 18: Reversible adiabatic transformation, work and ΔU

Exercise 19: Polytropic transformation, work and heat

Exercise 20: Adiabatic compression, final temperature and volume

Exercise 21: Polytropic transformation, calculation of k, W, Q

Exercise 22: Reversible cycle with isotherm, isobar, isochore

Exercise 23: Cycle with two isobars and two isochores

Exercise 24: Internal energy and isoenergetic transformation (Van der Waals)

Second Law and Entropy

Exercise 34: Heat exchange between two systems, entropy calculation

Exercise 35: Irreversible cooling, exchanged and created entropy

Exercise 36: Entropy variation with $C_P(T)$ depending on T

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Heat Transfer and Conduction

Exercise 16: Modes of heat transfer, Fourier's law, heat equation

Exercise 17: Heat equation in spherical symmetry, steady state

Exercise 18: Thermal balance in a rod with lateral losses

Kinetic Theory of Gases and Statistics

Exercise 19: Maxwell Boltzmann distribution, root-mean-square speed, internal energy

Phase Changes and Calorimetry

Exercise 24: Heating from ice to steam, latent heats and heat capacities

Exercise 25: Water-ice mixture, equilibrium temperature

Exercise 26: Mixtures, heat input by resistor, vaporization

Thermodynamic Cycles and Engines

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Exercice 33 Cycle with two isobars and two isochores

Exercice 10 (Cycle part)

Exercice 21 (Cycle)

Exact Differentials and State Functions

9, 10, 15, 26, 36 (Already classified)

Exercices and solutions

0.0.1 Exercise 1

Given $R = 8.31\,\mathrm{SI}$.

1. What is the equation of state for n moles of an ideal gas in the state P, V, T? Deduce the unit of R.

2. Numerically calculate the molar volume of an ideal gas at a pressure of 1 bar and a temperature of 0°C. Given: $1\,\mathrm{bar} = 10^5\,\mathrm{Pa}$.

0.0.2 Solution to Exercise 1

The state equation of an ideal gas is:

$$PV = nRT$$

The unit of R is:

$$[R] = \frac{[P][V]}{[n][T]} \equiv \mathrm{Pa} \cdot \mathrm{m}^3 \cdot \mathrm{mol}^{-1} \cdot \mathrm{K}^{-1}$$

The molar volume of an ideal gas at a pressure of 1 bar and a temperature of 0°C is:

$$V_0 = \frac{RT}{P} = \frac{8.31 \times (273.15 + 0)}{10^5} \approx 0.023\,\mathrm{m}^3$$

where $T(\mathrm{K}) = T(^\circ\mathrm{C}) + 273.15$.

0.0.3 Exercise 2

Let v be the specific volume in $\mathrm{m}^3 \cdot \mathrm{kg}^{-1}$ of an ideal gas with molar mass M.

1. Show that the equation of state for this gas can be written as $Pv = rT$. Specify the expression for r and its unit.

2. Given: $M(\mathrm{O}) = 16\,\mathrm{g} \cdot \mathrm{mol}^{-1}$, $R = 8.31\,\mathrm{SI}$, $1\,\mathrm{bar} = 10^5\,\mathrm{Pa}$. Calculate the value of r for dioxygen.

3. Deduce the specific volume of dioxygen at $300\,\mathrm{K}$ and $1\,\mathrm{bar}$.

0.0.4 Solution to Exercise 2

The specific volume v is defined as:

$$v = \frac{V}{m}$$

expressed in $\mathrm{m}^3 \cdot \mathrm{kg}^{-1}$. The number of moles is $n = \frac{m}{M}$.

From the ideal gas law:

$$PV = nRT \Rightarrow Pv = \frac{RT}{M} = rT$$

where $r = \frac{R}{M}$ and its unit is $\mathrm{kg}^{-1} \cdot \mathrm{Pa} \cdot \mathrm{m}^3 \cdot \mathrm{K}^{-1}$.

Calculation of r:

$$r = \frac{8.31}{32 \times 10^{-3}} \approx 259.69\,\mathrm{kg}^{-1} \cdot \mathrm{Pa} \cdot \mathrm{m}^3 \cdot \mathrm{K}^{-1}$$

The specific volume of dioxygen is:

$$v = \frac{rT}{P} = \frac{259.69 \times 300}{10^5} \approx 0.78\,\mathrm{m}^3$$

0.0.5 Exercise 3

A gas obeys the ideal gas equation. Starting from an equilibrium state of the gas, the pressure increases by 1% and the temperature by 2%. Determine the relative change in volume.

0.0.6 Solution to Exercise 3

From the ideal gas law:

$$PV = nRT \Rightarrow PdV + VdP = nRdT$$

Dividing by $PV = nRT$, we obtain:

$$\frac{dV}{V} + \frac{dP}{P} = \frac{dT}{T}$$

Thus, the relative variation of volume is:

$$\frac{dV}{V} = \frac{dT}{T} - \frac{dP}{P} = 2 - 1 = 1\%$$

0.0.7 Exercise 4

A gas obeys the Van der Waals equation, which for one mole is:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where a and b are positive constants.

1. In the SI system, what are the units of a and b?
2. Write the Van der Waals equation for n moles.

0.0.8 Solution to Exercise 4

The units of a and b are:

$$[P] = \left[\frac{a}{V^2}\right] \Rightarrow [a] = [P][V^2]$$

Thus, the unit of a is $\mathrm{Pa}\cdot\mathrm{m}^6$ and the unit of b is m^3.

Temperature and pressure are intensive parameters and do not depend on the number of moles, whereas volume is an extensive parameter. If we adopt the following notation:

1 mole	n moles
Temperature T_0	$T = T_0$
Pressure P_0	$P = P_0$
Volume V_0	$V = nV_0$

The Van der Waals equation for n moles is:

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

0.0.9 Exercise 5

Consider a thermodynamic system with the equation of state $F(P, V, T) = 0$.

1. Show that:
$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$
2. By calculating all partial derivatives, verify the formula for n moles of an ideal gas and a Van der Waals gas.

0.0.10 Solution to Exercise 5

A thermodynamic system with the state equation:

$$F(P,V,T) = 0 \Rightarrow \begin{cases} V = f(T,P) \Rightarrow dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP & (1) \\ P = g(T,V) \Rightarrow dP = \left(\frac{\partial P}{\partial T}\right)_V dT + \left(\frac{\partial P}{\partial V}\right)_T dV & (2) \\ T = h(V,P) \Rightarrow dT = \left(\frac{\partial T}{\partial V}\right)_P dV + \left(\frac{\partial T}{\partial P}\right)_V dP & (3) \end{cases}$$

We show that:

$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$

By substituting dT (equation (3)) into the relation for dV (equation (1)), we obtain:

$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP = \left(\frac{\partial V}{\partial T}\right)_P \left[\left(\frac{\partial T}{\partial V}\right)_P dV + \left(\frac{\partial T}{\partial P}\right)_V dP\right] + \left(\frac{\partial V}{\partial P}\right)_T dP$$

$$= \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial V}\right)_P dV + \left[\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V + \left(\frac{\partial V}{\partial P}\right)_T\right] dP$$

Since the elementary variation dV of the system is identical to itself, we obtain:

$$\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial V}\right)_P = 1 \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = -\frac{1}{\left(\frac{\partial T}{\partial V}\right)_P}$$

$$\left[\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V + \left(\frac{\partial V}{\partial P}\right)_T\right] = 0 \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -\left(\frac{\partial V}{\partial P}\right)_T = -\frac{1}{\left(\frac{\partial V}{\partial P}\right)_T}$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial V}\right)_T = -1$$

For n moles of an ideal gas $PV = nRT$, we calculate the derivatives:

$$\begin{cases} \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial\left[\frac{nRT}{P}\right]}{\partial T}\right)_P = \frac{nR}{P} \\ \left(\frac{\partial T}{\partial P}\right)_V = \left(\frac{\partial\left[\frac{PV}{nR}\right]}{\partial P}\right)_V = \frac{V}{nR} \\ \left(\frac{\partial P}{\partial V}\right)_T = \left(\frac{\partial\left[\frac{nRT}{V}\right]}{\partial V}\right)_T = -\frac{nRT}{V^2} \end{cases}$$

Thus:

$$\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial V}\right)_T = \frac{nR}{P} \cdot \frac{V}{nR} \cdot \left(-\frac{nRT}{V^2}\right) = -\frac{nRT}{PV} = -1$$

This relation is verified for ideal gases.

For n moles of a Van der Waals gas:

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

Differentiating this expression with respect to T (at constant P):

$$\frac{\partial}{\partial T}\left[P(V - nb) + \frac{an^2}{V^2}(V - nb) = nRT\right]_P$$

$$\Rightarrow P\left(\frac{\partial V}{\partial T}\right)_P - \frac{2an^2}{V^3}(V - nb)\left(\frac{\partial V}{\partial T}\right)_P + \frac{an^2}{V^2}\left(\frac{\partial V}{\partial T}\right)_P = nR$$

$$\Rightarrow \left[P - \frac{2an^2}{V^3}(V - nb) + \frac{an^2}{V^2}\right]\left(\frac{\partial V}{\partial T}\right)_P = nR$$

Substituting $P + \frac{an^2}{V^2} = \frac{nRT}{V-nb}$ into this expression, we obtain:

$$\left[-\frac{2an^2}{V^3}(V - nb) + \frac{nRT}{V - nb}\right]\left(\frac{\partial V}{\partial T}\right)_P = nR$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \frac{nR}{V - nb} \times \frac{1}{\frac{nRT}{(V-nb)^2} - \frac{2an^2}{V^3}}$$

Differentiating with respect to P (at constant V):

$$\frac{\partial}{\partial P}\left[P(V - nb) + \frac{an^2}{V^2}(V - nb) = nRT\right]_V$$

$$\Rightarrow (V-nb)+0+0+0=nR\left(\frac{\partial T}{\partial P}\right)_V$$

$$\Rightarrow \left(\frac{\partial T}{\partial P}\right)_V=\frac{V-nb}{nR}$$

Differentiating with respect to V (at constant T):

$$\frac{\partial}{\partial V}\left[P(V-nb)+\frac{an^2}{V^2}(V-nb)=nRT\right]_T$$

$$\Rightarrow (V-nb)\left(\frac{\partial P}{\partial V}\right)_T+P-\frac{2an^2}{V^3}(V-nb)+\frac{an^2}{V^2}=0$$

$$\Rightarrow \left(\frac{\partial P}{\partial V}\right)_T=\frac{2an^2}{V^3}-\frac{an^2}{V^2(V-nb)}-\frac{P}{V-nb}$$

$$\Rightarrow \left(\frac{\partial P}{\partial V}\right)_T=\frac{2an^2}{V^3}-\frac{nRT}{(V-nb)^2}$$

Thus:

$$\left(\frac{\partial V}{\partial T}\right)_P\left(\frac{\partial T}{\partial P}\right)_V\left(\frac{\partial P}{\partial V}\right)_T=-1$$

This relation is also verified for a Van der Waals gas.

0.0.11 Exercise 6

1. Calculate the thermodynamic coefficients α, β, χ for an ideal gas.
2. What must be the nature of the equation of state of a gas for $\alpha=\beta$?

0.0.12 Solution to Exercise 6

The thermodynamic coefficients α, β, and χ for an ideal gas are:

$$\alpha=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P=\frac{1}{V}\left(\frac{\partial\left[\frac{nRT}{P}\right]}{\partial T}\right)_P=\frac{nR}{PV}=\frac{1}{T}$$

$$\beta=\frac{1}{P}\left(\frac{\partial P}{\partial T}\right)_V=\frac{1}{P}\left(\frac{\partial\left[\frac{nRT}{V}\right]}{\partial T}\right)_V=\frac{nR}{PV}=\frac{1}{T}$$

$$\chi=-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T=-\frac{1}{V}\left(\frac{\partial\left[\frac{nRT}{P}\right]}{\partial P}\right)_T=\frac{nRT}{PV}=1$$

Thus, for ideal gases, we have $\alpha=\beta=\frac{1}{T}$ and $\chi=1$.

The nature of the state equation of a gas such that $\alpha=\beta$ is:

$$\alpha=\beta\Rightarrow\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P=\frac{1}{P}\left(\frac{\partial P}{\partial T}\right)_V$$

$$\Rightarrow \frac{P}{V} = \frac{\left(\frac{\partial P}{\partial T}\right)_V}{\left(\frac{\partial V}{\partial T}\right)_P} = \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P = -\frac{1}{\left(\frac{\partial V}{\partial P}\right)_T} = -\left(\frac{\partial P}{\partial V}\right)_T$$

$$\Rightarrow PdV = -VdP \Rightarrow \frac{dV}{V} = -\frac{dP}{P} \Rightarrow \ln V = -\ln P + \text{const} \Rightarrow \ln(PV) = \text{const} \Rightarrow PV = \text{const}$$

The nature of the gas at constant temperature is that of an ideal gas ($PV = nRT = \text{const}$).

0.0.13 Exercise 7

The equation of state of a fluid can be written as:

$$\ln \frac{V}{V_0} = a(T - T_0) - k(P - P_0)$$

1. Calculate the isobaric expansion coefficient α of this fluid.
2. What does the coefficient k represent?

0.0.14 Solution to Exercise 7

The state equation of a fluid is:

$$\ln \frac{V}{V_0} = a(T - T_0) - k(P - P_0)$$

The isobaric expansion coefficient α of this fluid is:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = a$$

The coefficient k is:

$$k = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = \chi$$

Thus, k represents the isothermal compressibility coefficient χ.

0.0.15 Exercise 8

A piece of metal is taken at 20°C under a pressure of 1 bar. Determine the pressure that must be applied to this piece of metal so that its volume remains constant when its temperature rises to 30°C. Given: $\alpha = 5 \times 10^{-5}\,\text{K}^{-1}$ and $\chi = 7 \times 10^{-12}\,\text{Pa}^{-1}$.

0.0.16 Solution to Exercise 8

Given the data α and χ, we have:

$$\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial V}\right)_T = -1$$

$$\Rightarrow \alpha V \left(\frac{\partial T}{\partial P}\right)_V \left(-\frac{1}{\chi V}\right) = -1$$

$$\Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha}{\chi}$$

$$\Rightarrow \chi dP = \alpha dT \Rightarrow \chi(P - P_0) = \alpha(T - T_0)$$

$$\Rightarrow P = \frac{\alpha}{\chi}(T - T_0) + P_0$$

Numerical application:

$$P = \frac{5 \times 10^{-5}}{7 \times 10^{-12}}(30 - 20) + 1 = \frac{5}{7} \times 10^8 + 1 \approx 7.15 \times 10^7\,\text{Pa} \approx 715\,\text{bar}$$

0.0.17 Exercise 9

Consider the differential:

$$dX = CdT + RT\frac{dV}{V}$$

where C and R are constants.

1. Is dX an exact differential?
2. Let $dS = g(T)dX$, with $g(T) = T^n$, where n is a positive or negative integer. What is the value of n for dS to be an exact differential?
3. Express in the case:
   (a) The partial derivatives $\left(\frac{\partial S}{\partial T}\right)_V$ and $\left(\frac{\partial S}{\partial V}\right)_T$.
   (b) The function $S(T, V)$ up to a constant.

0.0.18 Solution to Exercise 9

Consider the differential:

$$dX = CdT + RT\frac{dV}{V}$$

where C and R are constants.

Since dX varies with T and V, we have:

$$dX = \left(\frac{\partial X}{\partial T}\right)_V dT + \left(\frac{\partial X}{\partial V}\right)_T dV$$

For dX to be an exact differential, it must satisfy the following condition:

$$\frac{\partial}{\partial V}\left(\frac{\partial X}{\partial T}\right)_V = \frac{\partial}{\partial T}\left(\frac{\partial X}{\partial V}\right)_V$$

Calculating these derivatives:

$$\frac{\partial}{\partial V}\left(\frac{\partial X}{\partial T}\right)_V = \frac{\partial C}{\partial V} = 0$$

$$\frac{\partial}{\partial T}\left(\frac{\partial X}{\partial V}\right)_V = \frac{\partial}{\partial T}\left(\frac{RT}{V}\right)_V = \frac{R}{V}$$

Since $0 \neq \frac{R}{V}$, dX is not an exact differential.

Given:

$$\begin{cases} dS = g(T) \cdot dX \\ g(T) = T^n \\ dX = CdT + RT\frac{dV}{V} \end{cases}$$

$$\Rightarrow dS = T^n \cdot (CdT + \frac{RT}{V}dV) = CT^n dT + \frac{RT^{n+1}}{V}dV$$

For dS to be an exact differential:

$$\frac{\partial}{\partial V}(CT^n)_T = \frac{\partial}{\partial T}\left(\frac{RT^{n+1}}{V}\right)_V$$

$$\Rightarrow 0 = \frac{R}{V}(n+1)T^n$$

$$\Rightarrow n + 1 = 0 \Rightarrow n = -1$$

Thus:

$$dS = \frac{C}{T}dT + \frac{R}{V}dV$$

Integrating:

$$S(T, V) = C \ln T + R \ln V + C_0$$

where C_0 is an integration constant.

0.0.19 Exercise 10

Consider the two differentials:

$$dZ_1 = 2xy\,dx + x^2\,dy \quad \text{and} \quad dZ_2 = 2xy\,dx + xy\,dy$$

1. Are these differentials exact or inexact?
2. Calculate $\Delta Z = Z(1,1) - Z(0,0)$ for each differential and for each of the following paths:
   (a) Along the line $y = x$.
   (b) Along the curve $y = x^2$.

0.0.20 Solution to Exercise 10

Given:

$$dZ_1 = 2xy\,dx + x^2\,dy$$

$$dZ_2 = 2xy\,dx + xy\,dy$$

For dZ_1 to be an exact differential:

$$\frac{\partial}{\partial y}(2xy) = \frac{\partial}{\partial x}(x^2)$$

$$\Rightarrow 2x = 2x$$

Thus, dZ_1 is an exact differential, and Z_1 is a state function.

For dZ_2 to be an exact differential:

$$\frac{\partial}{\partial y}(xy) = \frac{\partial}{\partial x}(2xy)$$

$$\Rightarrow x \neq 2x$$

Thus, dZ_2 is not an exact differential, and Z_2 is not a state function.

Calculating $\Delta Z = Z(1,1) - Z(0,0)$ for each differential along the path $y = x$:

$$dZ_1 = dZ_2 = 3x^2\,dx$$

$$\Delta Z_1 = 3\int_0^1 x^2\,dx = 1$$

$$\Delta Z_2 = 3\int_0^1 x^2\,dx = 1$$

Along the path $y = x^2$:

$$dZ_1 = 4x^3\,dx$$

$$\Delta Z_1 = 4\int_0^1 x^3\,dx = 1$$

$$dZ_2 = 2x^3\,dx + 2x^4\,dx$$

$$\Delta Z_2 = 2\int_0^1 x^3\,dx + 2\int_0^1 x^4\,dx = \frac{1}{2} + \frac{2}{5} = \frac{9}{10}$$

0.0.21 Exercise 11

Consider a reversible elementary transformation for which a single-phase system is subjected only to pressure forces. The infinitesimal heat exchange δQ can be expressed as:

- Independent variables (T, V): $\delta Q = C_v dT + l dV$
- Independent variables (T, P): $\delta Q = C_p dT + h dP$
- Independent variables (P, V): $\delta Q = \lambda dP + \mu dV$

Express the coefficients l, h, λ, and μ in terms of the heat capacities C_p and C_v and the appropriate partial derivatives.

0.0.22 Solution to Exercise 11

Consider a reversible elementary transformation for a single-phase system subjected only to pressure forces. The infinitesimal heat exchanged δQ can be expressed as:

$$\delta Q(T, V) = C_v dT + l dV = \left(\frac{\partial Q}{\partial T}\right)_V dT + \left(\frac{\partial Q}{\partial V}\right)_T dV$$

$$\delta Q(T,P) = C_p dT + h dP = \left(\frac{\partial Q}{\partial T}\right)_P dT + \left(\frac{\partial Q}{\partial P}\right)_T dP$$

$$\delta Q(P,V) = \lambda dP + \mu dV = \left(\frac{\partial Q}{\partial P}\right)_V dP + \left(\frac{\partial Q}{\partial V}\right)_P dV$$

Since T is a state function:

$$dT = \left(\frac{\partial T}{\partial P}\right)_V dP + \left(\frac{\partial T}{\partial V}\right)_P dV$$

Substituting dT into the expressions for heat, we obtain:

$$\delta Q(T,V) = C_v \left(\frac{\partial T}{\partial P}\right)_V dP + \left[l + C_v \left(\frac{\partial T}{\partial V}\right)_P\right] dV$$

$$\delta Q(T,P) = \left[h + C_p \left(\frac{\partial T}{\partial P}\right)_V\right] dP + C_p \left(\frac{\partial T}{\partial V}\right)_P dV$$

$$\delta Q(P,V) = \lambda dP + \mu dV$$

Comparing these expressions, we find:

$$l + C_v \left(\frac{\partial T}{\partial V}\right)_P = C_p \left(\frac{\partial T}{\partial V}\right)_P \Rightarrow l = (C_p - C_v)\left(\frac{\partial T}{\partial V}\right)_P$$

$$C_v \left(\frac{\partial T}{\partial P}\right)_V = h + C_p \left(\frac{\partial T}{\partial P}\right)_V \Rightarrow h = (C_v - C_p)\left(\frac{\partial T}{\partial P}\right)_V$$

$$\lambda = C_v \left(\frac{\partial T}{\partial P}\right)_V$$

$$\mu = C_p \left(\frac{\partial T}{\partial V}\right)_P$$

0.0.23 Exercise 12

1. Consider a reversible transformation of an ideal gas. During an elementary step, the internal energy of the gas changes by dU; it receives the elementary work δW and the elementary heat δQ.
2. Express dU and δW and deduce δQ.
3. Show that δQ is not an exact differential and therefore heat is not a state function.

0.0.24 Solution to Exercise 12

For an ideal gas, the internal energy depends only on temperature:

$$dU = C_v dT$$

The elementary work is:

$$\delta W = -P dV$$

Thus, the heat exchanged is:

$$\delta Q = dU - \delta W = C_v dT + PdV$$

For δQ to be an exact differential, it must satisfy:

$$\left(\frac{\partial}{\partial V}\left(\frac{\partial Q}{\partial T}\right)_V\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{\partial Q}{\partial V}\right)_T\right)_V$$

$$\Rightarrow \left(\frac{\partial C_v}{\partial V}\right)_T \neq \left(\frac{\partial P}{\partial T}\right)_V$$

Thus, δQ is not an exact differential, and heat is not a state function.

0.0.25 Exercise 13

An ideal gas is contained in a cylinder closed by a piston. The walls of the cylinder and the piston are infinitely permeable to heat so that the studied transformations are isothermal. The initial conditions are P_0, V_0, T_0.

1. The gas is reversibly compressed from P_0 to P_1. Calculate the work W_1 exchanged during the operation. What is the work W_1' exchanged by the gas when it reversibly expands from P_1 to P_0? Compare W_1 and W_1' (sign and value).
2. The gas is irreversibly compressed from P_0 to P_1 by abruptly applying the pressure P_1 on the external face of the piston. Calculate the work W_2 exchanged during this operation. What is the work W_2' exchanged by the gas when it expands from P_1 to P_0, letting the pressure P_0 act on the external face of the piston? Compare W_2 and W_2'. Compare W_1 and W_2.

Take $P_0 = 1\,\text{atm}$, $P_1 = 10\,\text{atm}$.

0.0.26 Solution to Exercise 13

Given: $P_0 = 1\,\text{atm}$, $P_1 = 10\,\text{atm}$.

The work W_1 exchanged during the reversible compression from P_0 to P_1 is:

$$W_1 = -\int_{V_0}^{V_1} PdV = -nRT_0 \ln\frac{V_0}{V_1} = nRT_0 \ln\frac{P_1}{P_0}$$

The work W_1' exchanged during the reversible expansion from P_1 to P_0 is:

$$W_1' = -\int_{V_1}^{V_0} PdV = nRT_0 \ln\frac{V_1}{V_0} = nRT_0 \ln\frac{P_0}{P_1} = -W_1$$

For irreversible compression:

$$W_2 = P_1(V_0 - V_1) = nRT_0 P_1\left(\frac{1}{P_0} - \frac{1}{P_1}\right) = 9nRT_0$$

For irreversible expansion:

$$W'_2 = P_0(V_1 - V_0) = nRT_0P_0\left(\frac{1}{P_1} - \frac{1}{P_0}\right) = -0.9nRT_0$$

In absolute value, $W_2 > W'_2$ and they have opposite signs. Also, $W_1 = nRT_0 \ln 10 \approx 2.3nRT_0$ and $W_2 = 9nRT_0$, so $W_1 < W_2$.

0.0.27 Exercise 14

Calculate the change in internal energy for each of the following systems:

1. A system absorbs $Q = 2\,\mathrm{kJ}$ while it provides $W = 500\,\mathrm{J}$ of work to the surroundings.
2. A gas maintained at constant volume releases $Q = 5\,\mathrm{kJ}$.
3. The adiabatic compression of a gas is accomplished by $W = 80\,\mathrm{J}$ of work.

0.0.28 Solution to Exercise 14

Calculation of the internal energy variation for each system:

$$dU = \delta W + \delta Q \Rightarrow U = 2000 - 500 = 1500\,\mathrm{J} = 1.5\,\mathrm{kJ}$$

At constant volume:

$$\delta W = -PdV = 0 \Rightarrow U = Q = 5\,\mathrm{kJ}$$

For adiabatic compression:

$$\delta Q = 0 \Rightarrow U = W = 80\,\mathrm{J}$$

0.0.29 Exercise 15

The state variables pressure P, volume V, and temperature T are related by an equation of the form $f(P, V, T) = 0$, called the equation of state. The equation of state can also be written, for example, as $P = P(V, T)$, where the state variable P appears as a function of the independent variables V and T. Consider the total differentials:

$$dU = C_V dT + (l - P)dV \quad \text{and} \quad dS = \frac{C_V}{T}dT + \frac{l}{T}dV$$

where C_V and l are calorimetric coefficients that depend on the state variables.

1. Explain the relations imposed by the fact that dU and dS are exact differentials.
2. Consider the equation of state $PV - nRT = 0$, where n and R are constants. Calculate l; show that C_V does not depend on V, and calculate the state functions U and S, assuming that C_V does not depend on T.

0.0.30 Solution to Exercise 15

Given the total differentials:

$$dU = C_V dT + (l - P)dV$$

$$dS = \frac{C_V}{T}dT + \frac{l}{T}dV$$

where C_V and l are calorimetric coefficients depending on state variables.

For dU and dS to be exact differentials:

$$\left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial l}{\partial T}\right)_V - \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial\left[\frac{C_V}{T}\right]}{\partial V}\right)_T = \left(\frac{\partial\left[\frac{l}{T}\right]}{\partial T}\right)_V$$

For an ideal gas $PV = nRT$:

$$l = P$$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = 0$$

Thus, C_V does not depend on V.

The internal energy U is:

$$U = C_V T + \text{const}$$

The entropy S is:

$$S = C_V \ln T + nR \ln V + \text{const}$$

0.0.31 Exercise 16

1. What are the different processes of heat transfer?
   as a thermostat, and on the outer face of the wall.

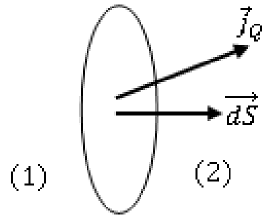


Figure 1: Elementary surface dS.

2. To describe thermal energy transfers, the heat flux density vector $\vec{J_Q}$ is sometimes introduced, defined by its flux $d\phi$ through an elementary surface $\vec{ds}$ oriented from (1) to (2): the elementary flux $d\phi = \vec{J_Q} \cdot \vec{ds}$ represents the thermal power exchanged through $\vec{ds}$, provided by 1 and received by 2. The flux $d\phi$ being an algebraic quantity, explain the direction of the effective power transfers between (1) and (2) through $\vec{ds}$ when $d\phi$ is positive, and then when $d\phi$ is negative.

3. The mathematical expression of $\vec{J_Q}$ in the case of heat transfer through a solid medium is of the form:

$$\vec{J_Q} = -\lambda \vec{\nabla} T$$

   where λ represents the thermal conductivity of the medium ($\lambda > 0$). Recall the expression in Cartesian coordinates of the gradient operator $\vec{\nabla}$:

$$\vec{\nabla} T(x, y, z, t) = \frac{\partial T}{\partial x}\vec{e_x} + \frac{\partial T}{\partial y}\vec{e_y} + \frac{\partial T}{\partial z}\vec{e_z}$$

   (a) Explain the significance of the minus sign in the expression of $\vec{J_Q}$.

   (b) Recall the unit of λ.

4. Consider a solid of thermal conductivity λ, to which the frame $(O, \vec{i}, \vec{j}, \vec{k})$ is attached. Its specific heat is denoted c, its mass density is denoted ρ. The quantities λ, ρ, and c are assumed to be uniform and stationary. Inside the solid, the temperature T is a function of the position variable x and time t. We denote this function $T(x, t)$.

   (a) With these assumptions, give the simplified expression of $\vec{J_Q}$ as a function of λ, a partial derivative of T, and a unit basis vector.

   (b) Consider the portion of the solid below: as a thermostat, and on the outer face of the wall. The portion of the solid considered is delimited by the closed surface consisting of the surfaces S_1, S_2, and S_l. The points of S_1 all have the same abscissa x, those of S_2 have abscissa $x + dx$. Let S be the area of the surfaces S_1 and S_2. The surface S_l is a portion of a cylinder of height dx whose generatrices are parallel to the axis (Ox). Assume also that there is no heat source in the solid.

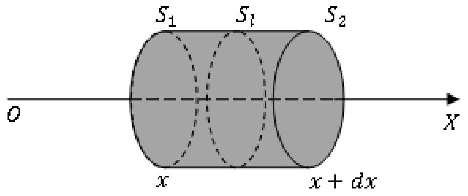


Figure 2: Heat transfer.

      i. Let δQ_1 be the elementary heat received by the portion of the solid through the surface S_1 between time t and $t + dt$. Express δQ_1 as a function of λ, S, dt, and a partial derivative of T. Be sure to specify the values of the variables of the partial derivative.

      ii. Similarly, δQ_2 represents the elementary heat given by the portion of the solid through the surface S_2 between time t and $t + dt$. Express δQ_2 as a function of λ, S, dt, and a partial derivative of T. Be sure to specify the values of the variables of the partial derivative.

      iii. Justify that the portion of the solid does not exchange any thermal energy through S_l.

iv. Let $\delta^2 Q$ be the total heat received by the portion of the solid between times t and $t + dt$. Relate $\delta^2 Q$ to δQ_1 and δQ_2. Then express $\delta^2 Q$ as a function of a derivative of T, λ, S, dt, and dx.

v. Also relate $\delta^2 Q$ to the change in temperature dT of the portion studied between these two times. Involve the quantities ρ, c, S, and dx.

vi. Express the change in temperature dT between times t and $t + dt$ as a function of a partial derivative of $T(x, t)$ and dt.

vii. Deduce from the three previous questions that the following differential equation is obtained:

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}$$

Specify the expression of a as a function of the mass density ρ, the specific heat c, and the thermal conductivity λ. This relation constitutes the heat equation in one dimension. The coefficient a is called the thermal diffusivity.

0.0.32 Solution to Exercise 16

1. **Different processes of heat transfer:**
The three main processes of heat transfer are:

- Conduction: Transfer of heat through a solid medium without the movement of matter.

- Convection: Transfer of heat through fluids (liquids or gases) involving the movement of matter.

- Radiation: Transfer of heat via electromagnetic waves, without requiring any medium.

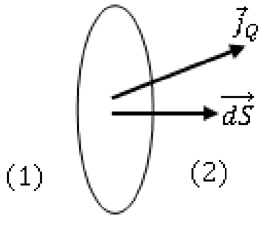


Figure 3: Elementary surface dS.

2. **Direction of power transfers based on $d\phi$:**
The elementary flux $d\phi = \vec{J_Q} \cdot \vec{ds}$ represents the thermal power exchanged through the surface element $\vec{ds}$, oriented from (1) to (2):

- If $d\phi > 0$, the thermal power flows from region (1) to region (2).

- If $d\phi < 0$, the thermal power flows from region (2) to region (1).

3. **Heat flux density vector and gradient operator:**

   (a) **Significance of the minus sign in $\vec{J_Q} = -\lambda\vec{\nabla}T$:**
   The minus sign indicates that heat flows from regions of higher temperature to regions of lower temperature, as per Fourier's law. This is consistent with the second law of thermodynamics, which states that energy spontaneously flows from hot to cold regions.

   (b) **Unit of λ:**
   The thermal conductivity λ has units of $\mathrm{Wm^{-1}K^{-1}}$ (watts per meter-kelvin).

4. **Heat transfer in a solid:**

   (a) **Simplified expression of $\vec{J_Q}$:**
   Since $T(x,t)$ depends only on x and t, the gradient simplifies to:

$$\vec{\nabla}T = \frac{\partial T}{\partial x}\vec{e_x}.$$

   Thus, the heat flux density vector becomes:

$$\vec{J_Q} = -\lambda\frac{\partial T}{\partial x}\vec{e_x}.$$

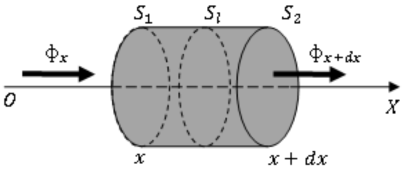


Figure 4: Elementary surface dS.

   (b) **Heat transfer analysis for the portion of the solid:**

      i. **Elementary heat received through S_1:**
      The heat flux through S_1 is given by:

$$\delta Q_1 = -\left(\vec{J_Q}\cdot\vec{n_1}\right)Sdt = \lambda S\left.\frac{\partial T}{\partial x}\right|_x dt,$$

      where $\vec{n_1} = -\vec{e_x}$ is the outward normal at S_1.

      ii. **Elementary heat given through S_2:**
      Similarly, the heat flux through S_2 is:

$$\delta Q_2 = -\left(\vec{J_Q}\cdot\vec{n_2}\right)Sdt = -\lambda S\left.\frac{\partial T}{\partial x}\right|_{x+dx} dt,$$

      where $\vec{n_2} = \vec{e_x}$ is the outward normal at S_2.

      iii. **No heat exchange through S_l:**
      The lateral surface S_l is adiabatic (no heat source), so no heat is exchanged through it.

iv. **Total heat received by the portion:**
The total heat received is:

$$\delta^2 Q = \delta Q_1 + \delta Q_2 = \lambda S \left(\left.\frac{\partial T}{\partial x}\right|_x - \left.\frac{\partial T}{\partial x}\right|_{x+dx} \right) dt.$$

Using a Taylor expansion for $\left.\frac{\partial T}{\partial x}\right|_{x+dx}$:

$$\left.\frac{\partial T}{\partial x}\right|_{x+dx} \approx \left.\frac{\partial T}{\partial x}\right|_x + dx \frac{\partial^2 T}{\partial x^2}.$$

Substituting this into $\delta^2 Q$:

$$\delta^2 Q = -\lambda S dx \frac{\partial^2 T}{\partial x^2} dt.$$

v. **Relation to temperature change:**
The heat received causes a temperature change dT in the portion of the solid:

$$\delta^2 Q = mcdT = (\rho S dx) c dT.$$

vi. **Change in temperature dT:**
The temperature change can be expressed as:

$$dT = \frac{\partial T}{\partial t} dt.$$

vii. **Derivation of the heat equation:**
Equating the two expressions for $\delta^2 Q$:

$$-\lambda S dx \frac{\partial^2 T}{\partial x^2} dt = (\rho S dx) c \frac{\partial T}{\partial t} dt.$$

Simplifying:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2},$$

where the thermal diffusivity a is:

$$a = \frac{\lambda}{\rho c}.$$

0.0.33 Exercise 17

A futuristic house consists of a hemispherical wall with inner radius R_1 and outer radius R_2, placed on a horizontal ground. Let O be the center of the complete sphere and M a point on the wall (thus such that $OM = r$ is between R_1 and R_2). The material constituting it is homogeneous and isotropic, with mass density μ and specific heat capacity C. A constant temperature T_1 prevails inside the house and on the inner face of the wall, and a temperature $T_2 < T_1$ prevails in the outside air, considered as a thermostat, and on the outer face of the wall.

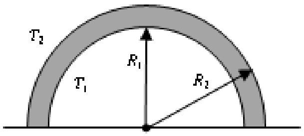


Figure 5: The futuristic house.

1. Write the partial differential equation satisfied by the temperature $T(r,t)$ at any point in the material. Assume invariance under any rotation around an axis passing through O. Recall the expression of the Laplacian of a scalar function $U(r,t)$ in spherical coordinates:

$$\Delta U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right)$$

2. In the stationary regime, determine the temperature inside the wall.
3. Express the heat flux escaping from the house, neglecting the effects of the ground.
4. What is the thermal resistance of the house, still neglecting the effects of the ground?

0.0.34 Solution to Exercise 17

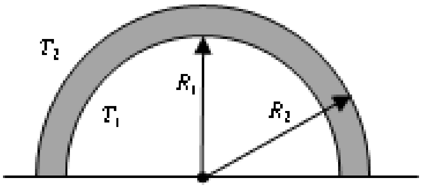


Figure 6: The futuristic house.

1. Partial Differential Equation for $T(r,t)$

The temperature $T(r,t)$ in the material satisfies the heat equation:

$$\frac{\partial T}{\partial t} = \alpha \Delta T,$$

where $\alpha = \frac{k}{\mu C}$ is the thermal diffusivity of the material, k is the thermal conductivity, μ is the mass density, and C is the specific heat capacity.

Assuming spherical symmetry (invariance under any rotation around an axis passing through O), the Laplacian in spherical coordinates simplifies to:

$$\Delta T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right).$$

Thus, the partial differential equation becomes:

$$\frac{\partial T}{\partial t} = \alpha\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right).$$

2. Temperature Distribution in the Stationary Regime

In the stationary regime, the temperature does not depend on time ($\frac{\partial T}{\partial t} = 0$). The heat equation reduces to:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0.$$

Multiplying through by r^2, we obtain:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0.$$

This implies that:

$$r^2\frac{\partial T}{\partial r} = C_1,$$

where C_1 is a constant of integration. Dividing by r^2, we get:

$$\frac{\partial T}{\partial r} = \frac{C_1}{r^2}.$$

Integrating again with respect to r, we find:

$$T(r) = -\frac{C_1}{r} + C_2,$$

where C_2 is another constant of integration.

To determine C_1 and C_2, we use the boundary conditions:

$$T(R_1) = T_1 \quad \text{and} \quad T(R_2) = T_2.$$

Substituting $r = R_1$ into the general solution:

$$T_1 = -\frac{C_1}{R_1} + C_2.$$

Substituting $r = R_2$:

$$T_2 = -\frac{C_1}{R_2} + C_2.$$

We solve this system of equations for C_1 and C_2. Subtracting the second equation from the first:

$$T_1 - T_2 = -\frac{C_1}{R_1} + \frac{C_1}{R_2}.$$

Factoring out C_1:

$$T_1 - T_2 = C_1 \left(\frac{1}{R_2} - \frac{1}{R_1} \right).$$

Solving for C_1:

$$C_1 = \frac{T_1 - T_2}{\frac{1}{R_2} - \frac{1}{R_1}} = \frac{(T_1 - T_2)R_1 R_2}{R_1 - R_2}.$$

Substituting C_1 back into one of the boundary conditions (e.g., $T_1 = -\frac{C_1}{R_1} + C_2$):

$$C_2 = T_1 + \frac{C_1}{R_1} = T_1 + \frac{\frac{(T_1 - T_2)R_1 R_2}{R_1 - R_2}}{R_1}.$$

Simplifying:

$$C_2 = T_1 + \frac{(T_1 - T_2)R_2}{R_1 - R_2}.$$

Thus, the temperature distribution in the stationary regime is:

$$T(r) = -\frac{(T_1 - T_2)R_1 R_2}{(R_1 - R_2)r} + T_1 + \frac{(T_1 - T_2)R_2}{R_1 - R_2}.$$

3. Heat Flux Escaping from the House

The heat flux density is given by Fourier's law:

$$\mathbf{q} = -k\nabla T.$$

In spherical symmetry, the radial component of the heat flux density is:

$$q_r = -k\frac{\partial T}{\partial r}.$$

From the expression for $T(r)$, we have:

$$\frac{\partial T}{\partial r} = \frac{(T_1 - T_2)R_1 R_2}{(R_1 - R_2)r^2}.$$

Thus:

$$q_r = -k\frac{(T_1 - T_2)R_1 R_2}{(R_1 - R_2)r^2}.$$

The total heat flux escaping from the house is obtained by integrating over the outer surface of the hemisphere ($r = R_2$):

$$\Phi = \int_{\text{hemisphere}} q_r \, dA.$$

The surface area element in spherical coordinates is $dA = 2\pi r^2 \sin\theta \, d\theta$, but for a hemisphere, the total surface area is simply:

$$A = 2\pi R_2^2.$$

Therefore:

$$\Phi = q_r(R_2) \cdot A = \left(-k \frac{(T_1 - T_2)R_1R_2}{(R_1 - R_2)R_2^2} \right) \cdot 2\pi R_2^2.$$

Simplifying:

$$\Phi = -2\pi k \frac{(T_1 - T_2)R_1R_2}{R_1 - R_2}.$$

Since $T_1 > T_2$, the negative sign indicates heat flow outward, so:

$$\boxed{\Phi = 2\pi k \frac{(T_1 - T_2)R_1R_2}{R_1 - R_2}.}$$

4. Thermal Resistance of the House

Thermal resistance R_{th} is defined as:

$$R_{\text{th}} = \frac{\Delta T}{\Phi}.$$

Here, $\Delta T = T_1 - T_2$ and $\Phi = 2\pi k \frac{(T_1 - T_2)R_1R_2}{R_1 - R_2}$. Substituting:

$$R_{\text{th}} = \frac{T_1 - T_2}{2\pi k \frac{(T_1 - T_2)R_1R_2}{R_1 - R_2}}.$$

Simplifying:

$$R_{\text{th}} = \frac{R_1 - R_2}{2\pi k R_1 R_2}.$$

Thus, the thermal resistance of the house is:

$$\boxed{R_{\text{th}} = \frac{R_1 - R_2}{2\pi k R_1 R_2}.}$$

0.0.35 Exercise 18

A solid copper rod, cylindrical, of axis (OX), length l, radius a, and thermal conductivity K, is in contact at one of its ends $(x = 0)$ with a heat exchanger at temperature T_0 and at its lateral surface and its other end $(x = l)$ it is in contact with a fluid at constant temperature T_e $(T_0 > T_e)$.

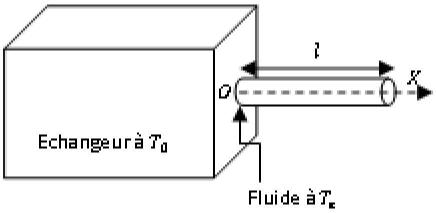


Figure 7: A copper rod.

1. We are in a steady state and assume that inside the rod the radial temperature gradient is sufficiently weak to consider that in the cross-section at abscissa x, the temperature $T(x)$ is uniform. The rod presents, at the level of its surface in contact with the fluid, thermal losses, per unit time and surface, equal to $h(T(x) - T_e)$, if $T(x)$ denotes the temperature of the point of the surface considered and h a constant coefficient.
2. Determine the temperature distribution $T(x)$ within the rod.
3. Calculate $T(l)$. Given: $K = 389\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$, $h = 155\,\mathrm{W \cdot m^{-2} \cdot K^{-1}}$, $a = 1\,\mathrm{mm}$, $T_0 = 340\,\mathrm{K}$, $T_e = 300\,\mathrm{K}$, and $l = 10\,\mathrm{cm}$.

0.0.36 Solution to Exercice 18

A solid copper rod, cylindrical in shape, with axis (OX), length l, radius a, and thermal conductivity K, is in contact at one of its ends ($x = 0$) with a heat exchanger at temperature T_0. Its lateral surface and the other end ($x = l$) are in contact with a fluid at constant temperature T_e ($T_0 > T_e$).

We assume steady-state conditions and that the radial temperature gradient inside the rod is sufficiently small to consider the temperature $T(x)$ uniform in the cross-section at abscissa x. The rod exhibits thermal losses on its surface in contact with the fluid, per unit time and surface area, equal to $h(T(x) - T_e)$, where $T(x)$ is the temperature at the considered point of the surface and h is a constant coefficient.

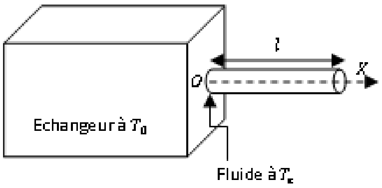


Figure 8: The solid copper rod.

1. Heat Balance Equation

The heat flux along the rod satisfies the following balance equation:

$$\phi_x = \phi_{x+dx} + h(T(x) - T_e) \cdot dS_{\text{lateral}},$$

where ϕ_x is the heat flux at position x, and $dS_{\text{lateral}} = 2\pi a\, dx$ is the lateral surface area of the rod over an infinitesimal length dx. Substituting the heat flux expressions:

$$j_x \pi a^2 = j_{x+dx} \pi a^2 + h[T(x) - T_e] \cdot 2\pi a\, dx,$$

with $j_x = -K\frac{\partial T(x)}{\partial x}$ and $j_{x+dx} = -K\frac{\partial T(x+dx)}{\partial x}$.

After simplification:

$$\pi a^2 (j_x - j_{x+dx}) = h[T(x) - T_e] \cdot 2\pi a\, dx.$$

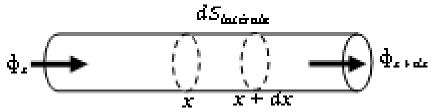


Figure 9: The sections.

Substituting j_x and j_{x+dx}:

$$\pi a^2 \left(-K \frac{\partial T(x)}{\partial x} + K \frac{\partial T(x+dx)}{\partial x} \right) = h[T(x) - T_e] \cdot 2\pi a \, dx.$$

Further simplification yields:

$$Ka \frac{\partial}{\partial x} \left(\frac{\partial T(x)}{\partial x} \right) = 2h[T(x) - T_e].$$

Thus, the second-order differential equation governing the temperature distribution is:

$$\frac{\partial^2 T(x)}{\partial x^2} = \frac{2h}{Ka} \left(T(x) - T_e \right). \tag{1}$$

2. Solving the Differential Equation

The solution process involves three steps:

Step 1: Homogeneous Solution For the homogeneous equation:

$$\frac{\partial^2 T(x)}{\partial x^2} = \frac{2h}{Ka} T(x),$$

the characteristic equation is:

$$r^2 - \frac{2h}{Ka} = 0 \quad \Rightarrow \quad r = \pm \sqrt{\frac{2h}{Ka}}.$$

The homogeneous solution is:

$$T(x) = Ae^{\sqrt{\frac{2h}{Ka}}x} + Be^{-\sqrt{\frac{2h}{Ka}}x}.$$

Step 2: Particular Solution Assume a constant solution $T(x) = C$. Substituting into Equation (1):

$$\frac{\partial^2 C}{\partial x^2} = \frac{2h}{Ka}(C - T_e) = 0 \quad \Rightarrow \quad C = T_e.$$

Thus, $T(x) = T_e$ is a particular solution.

Step 3: General Solution The general solution is the sum of the homogeneous and particular solutions:

$$T(x) = Ae^{\sqrt{\frac{2h}{Ka}}x} + Be^{-\sqrt{\frac{2h}{Ka}}x} + T_e.$$

3. Applying Boundary Conditions

Boundary Condition 1: At $x = 0$, $T(0) = T_0$:

$$T(0) = Ae^0 + Be^0 + T_e = T_0 \quad \Rightarrow \quad A + B = T_0 - T_e. \tag{2}$$

Boundary Condition 2: At $x = l$, $T(l) = T_e$:

$$T(l) = Ae^{\sqrt{\frac{2h}{Ka}}l} + Be^{-\sqrt{\frac{2h}{Ka}}l} + T_e = T_e.$$

Simplifying:

$$Ae^{\sqrt{\frac{2h}{Ka}}l} + Be^{-\sqrt{\frac{2h}{Ka}}l} = 0. \tag{3}$$

From Equations (2) and (3), solve for A and B:

$$A = \frac{(T_e - T_0)e^{-\sqrt{\frac{2h}{Ka}}l}}{e^{\sqrt{\frac{2h}{Ka}}l} - e^{-\sqrt{\frac{2h}{Ka}}l}}, \quad B = \frac{(T_0 - T_e)e^{\sqrt{\frac{2h}{Ka}}l}}{e^{\sqrt{\frac{2h}{Ka}}l} - e^{-\sqrt{\frac{2h}{Ka}}l}}.$$

Substitute A and B into the general solution:

$$T(x) = \frac{T_e - T_0}{e^{\sqrt{\frac{2h}{Ka}}l} - e^{-\sqrt{\frac{2h}{Ka}}l}} \left(e^{\sqrt{\frac{2h}{Ka}}(x-l)} - e^{\sqrt{\frac{2h}{Ka}}(l-x)} \right) + T_e.$$

4. Calculating $T(l)$

Using the given data:

$$K = 389\,\mathrm{W}\cdot\mathrm{m}^{-1}\cdot\mathrm{K}^{-1}, \quad h = 155\,\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-1}, \quad a = 1\,\mathrm{mm} = 0.001\,\mathrm{m},$$

$$T_0 = 340\,\mathrm{K}, \quad T_e = 300\,\mathrm{K}, \quad l = 10\,\mathrm{cm} = 0.1\,\mathrm{m}.$$

Compute $\alpha = \sqrt{\frac{2h}{Ka}}$:

$$\alpha^2 = \frac{2 \cdot 155}{389 \cdot 0.001} = \frac{310}{0.389} \approx 796.91 \quad \Rightarrow \quad \alpha \approx \sqrt{796.91} \approx 28.23\,\mathrm{m}^{-1}.$$

Substitute into the expression for $T(l)$ and compute numerically:

$$T(l) = \frac{-40}{e^{28.23\cdot 300} - e^{-28.23\cdot 300}} \left(e^{28.23\cdot(0.1-300)} - e^{28.23\cdot(300-0.1)} \right) + T_e.$$

After numerical evaluation:

$$\boxed{T(l) \approx 303.2\,\mathrm{K}}.$$

0.0.37 Exercise 19

Given:

$$\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}, \quad \int_0^\infty x^4 e^{-ax^2} dx = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}}$$

1. In the furnace, cesium 133 behaves as a monoatomic ideal gas. A volume V contains N atoms of mass m (i.e., $n_0 = N/V$ atoms per unit volume) at temperature T.

2. Indicate the expression of the kinetic energy E of an atom of a monoatomic ideal gas as a function of its velocity.

3. It is admitted that the previous gas obeys Boltzmann statistics: the number of atoms contained in an infinitesimal volume dV, whose velocity vectors have a norm between v and $v + dv$, is:

$$dN = A\exp\left(-\frac{E}{kT}\right)v^2dvdV$$

4. Give the expression of N, the total number of atoms contained in the volume V, as a function of A.

5. Determine the elementary probability $dP = f(v)dv$.

6. Calculate the root mean square velocity v_q (square root of the mean of the squares of the velocities of the various atoms) using the previous results.

7. Define the internal energy U of the studied ideal gas contained in the volume V; give its expression as a function of the absolute temperature T.

0.0.38 Solution to Exercice 19

$$\int_0^\infty x^2e^{-ax^2}\,dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}},\quad \int_0^\infty x^4e^{-ax^2}\,dx = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}}$$

In the oven, cesium-133 behaves like a monoatomic ideal gas. A volume V contains N atoms of mass m (i.e., $n_0 = \frac{N}{V}$ atoms per unit volume) at temperature T. 1- The expression for the kinetic energy E of an atom in a monoatomic ideal gas as a function of its velocity is the translational kinetic energy:

$$E = \frac{1}{2}mv^2,$$

where m is the mass of the atom and v is its velocity.

2- We assume that the gas follows Boltzmann statistics: the number of atoms contained in an infinitesimal volume dV, whose velocity magnitudes lie between v and $v + dv$, is:

$$dN = A\exp\left(-\frac{E}{kT}\right)v^2\,dv\,dV.$$

3- The total number of atoms N contained in the volume V as a function of A is:

$$N = \int_0^\infty A\exp\left(-\frac{E}{kT}\right)v^2\,dv\,V,$$

since the magnitude of the velocity v can vary between 0 and ∞.

Thus:

$$N = AV\int_0^\infty \exp\left(-\frac{mv^2}{2kT}\right)v^2\,dv.$$

This integral is of the form:

$$\int_0^\infty x^2 e^{-ax^2}\,dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}.$$

Therefore:

$$\int_0^\infty \exp\left(-\frac{mv^2}{2kT}\right) v^2\,dv = \frac{2kT}{4m}\sqrt{\frac{2kT\pi}{m}} = \frac{kT}{2m}\sqrt{\frac{2kT\pi}{m}}.$$

Hence:

$$N = AV\frac{kT}{2m}\sqrt{\frac{2\pi kT}{m}}.$$

4- The elementary probability is:

$$dP = f(v)\,dv = \frac{dN}{N} = \frac{AV\exp\left(-\frac{mv^2}{2kT}\right)v^2\,dv}{AV\frac{kT}{2m}\sqrt{\frac{2kT\pi}{m}}}.$$

Simplifying:

$$dP = f(v)\,dv = \frac{2m}{kT}\sqrt{\frac{m}{2\pi kT}}\exp\left(-\frac{mv^2}{2kT}\right)v^2\,dv.$$

Calculation of the Root Mean Square Velocity v_q

The root mean square velocity v_q (square root of the mean of the squares of the velocities of the various atoms) is calculated using the previous result:

$$v_q = \sqrt{\langle v^2\rangle} = \sqrt{\int_0^\infty v^2 f(v)\,dv} = \sqrt{\int_0^\infty \frac{2m}{kT}\sqrt{\frac{m}{2\pi kT}}\exp\left(-\frac{mv^2}{2kT}\right)v^4\,dv}.$$

This gives an integral of the form:

$$\int_0^\infty x^4 e^{-ax^2}\,dx = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}},$$

where $x = v$ and $a = \frac{m}{2kT}$. Substituting:

$$\int_0^\infty \exp\left(-\frac{mv^2}{2kT}\right)v^4\,dv = \frac{3}{8\left(\frac{m}{2kT}\right)^2}\sqrt{\frac{\pi}{\frac{m}{2kT}}} = \frac{3k^2T^2}{2m^2}\sqrt{\frac{2\pi kT}{m}}.$$

Thus:

$$v_q = \sqrt{\frac{2m}{kT}\sqrt{\frac{m}{2\pi kT}}\cdot\frac{3k^2T^2}{2m^2}\sqrt{\frac{2\pi kT}{m}}}.$$

Simplifying:

$$v_q = \sqrt{\frac{3kT}{m}}.$$

$$\boxed{v_q = \sqrt{\frac{3kT}{m}}.}$$

Internal Energy U of the Ideal Gas

The internal energy U of the ideal gas under study is the sum of the kinetic energies of the atoms contained in the volume V. On average, U is therefore:

$$U = \langle \frac{1}{2} m v^2 \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m v_q^2 = \frac{1}{2} m \cdot \frac{3kT}{m}.$$

Simplifying:

$$\boxed{U = \frac{3}{2} kT.}$$

This is the expression for the internal energy as a function of the absolute temperature T.

0.0.39 Exercise 20

One mole of Van der Waals gas, with the equation of state $(P + \frac{a}{V^2})(V - b) = RT$, undergoes a reversible isothermal transformation where its volume changes from V_1 to V_2.

- Calculate the work exchanged with the surroundings.
- Compare the result obtained with that of an ideal gas.

0.0.40 Solution to Exercise 20

The work exchanged with the surroundings is given by:

$$\delta W = -P \, dV$$

where the pressure P is:

$$P = \frac{RT}{V - b} - \frac{a}{V^2}$$

For an isothermal transformation ($T_2 = T_1 = T =$ constant):

$$\delta W = -\left(\frac{RT}{V - b} - \frac{a}{V^2} \right) dV$$

$$W_{12} = -\int_{V_1}^{V_2} \frac{RT}{V - b} \, dV + \int_{V_1}^{V_2} \frac{a}{V^2} \, dV$$

$$W_{12} = -RT \int_{V_1}^{V_2} \frac{dV}{V - b} + a \int_{V_1}^{V_2} \frac{dV}{V^2}$$

$$W_{12} = -RT \ln(V - b) \Big|_{V_1}^{V_2} - \frac{a}{V} \Big|_{V_1}^{V_2}$$

$$W_{12} = -RT \left[\ln(V_2 - b) - \ln(V_1 - b) \right] - a \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$$

$$W_{12} = RT \ln \left(\frac{V_1 - b}{V_2 - b} \right) + a \left(\frac{1}{V_1} - \frac{1}{V_2} \right)$$

Comparison with an ideal gas ($PV = RT$):

$$P = \frac{RT}{V}$$

$$W'_{12} = -\int_{V_1}^{V_2} \frac{RT}{V}\,dV = -RT\int_{V_1}^{V_2} \frac{dV}{V} = -RT\left(\ln V_2 - \ln V_1\right) = RT\ln\left(\frac{V_1}{V_2}\right)$$

$$W_{12} = RT\ln\left(\frac{V_1 - b}{V_2 - b}\right) + a\left(\frac{1}{V_1} - \frac{1}{V_2}\right) = RT\ln\left(\frac{V_1}{V_2}\cdot\frac{1-\frac{b}{V_1}}{1-\frac{b}{V_2}}\right) + a\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$

$$W_{12} = RT\ln\left(\frac{V_1}{V_2}\right) + RT\ln\left(\frac{1-\frac{b}{V_1}}{1-\frac{b}{V_2}}\right) + a\left(\frac{1}{V_1} - \frac{1}{V_2}\right) = W'_{12} + RT\ln\left(\frac{1-\frac{b}{V_1}}{1-\frac{b}{V_2}}\right) + a\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$

0.0.41 Exercise 21

One mole of an ideal gas, initially in state $A(P_1, V_1, T_1)$, undergoes an isobaric expansion to state $B(P_1, V_2, T_2)$, followed by an isochoric process to state $C(P_2, V_2, T_1)$, and finally an isothermal compression back to state A.

- Calculate the work done during each transformation.

0.0.42 Solution to Exercise 21

Transformation $A \to B$:

This transformation is isobaric, meaning the pressure is constant ($P = P_1$):

$$\delta W = -P\,dV \Rightarrow W_{AB} = -\int_{V_1}^{V_2} P\,dV = -P_1\int_{V_1}^{V_2} dV = -P_1(V_2 - V_1)$$

$$W_{AB} = P_1(V_1 - V_2)$$

Transformation $B \to C$:

This transformation is isochoric, meaning the volume is constant ($V = V_2$):

$$\delta W = -P\,dV \Rightarrow W_{BC} = 0$$

Transformation $C \to A$:

This transformation is isothermal, meaning the temperature is constant ($T = T_1$):
The equation of state for one mole of an ideal gas is $PV = RT_1$, so $P = \frac{RT_1}{V}$:

$$\delta W = -P\,dV \Rightarrow W_{CA} = -\int_{V_1}^{V_2} \frac{RT_1}{V}\,dV = -RT_1\int_{V_1}^{V_2} \frac{dV}{V} = -RT_1 \ln V\Big|_{V_1}^{V_2} = RT_1\ln\left(\frac{V_1}{V_2}\right)$$

0.0.43 Exercise 22

One mole of an ideal gas at an initial temperature of 298 K expands from an initial pressure of 5 atm to a final pressure of 1 atm. In each of the following cases:

- Reversible isothermal expansion.
- Irreversible isothermal expansion.

Calculate the final temperature T_2 of the gas and the work W done by the gas.

0.0.44 Solution to Exercise 22

Isothermal and Reversible Expansion:

For an isothermal expansion ($T = T_1 = T_2 = 298\,\mathrm{K}$): For one mole of an ideal gas, the equation of state is $PV = RT$, so $P = \frac{RT}{V}$. During a reversible transformation, the pressure changes and is variable:

$$\delta W = -P\,dV = -\frac{RT}{V}\,dV \Rightarrow W = -RT\int_{V_1}^{V_2}\frac{dV}{V} = -RT\ln V\Big|_{V_1}^{V_2} = RT\ln\left(\frac{V_1}{V_2}\right)$$

$$W = RT\ln\left(\frac{V_1}{V_2}\right)$$

Given:

$$P_1V_1 = RT_1, \quad P_2V_2 = RT_2, \quad T = T_1 = T_2$$

$$P_1V_1 = P_2V_2 \Rightarrow \frac{V_1}{V_2} = \frac{P_2}{P_1}$$

$$W = RT\ln\left(\frac{P_2}{P_1}\right)$$

Numerical application: $W = -3987\,\mathrm{J}$.

Isothermal and Irreversible Expansion:

In an irreversible transformation, the pressure is constant and equal to the final pressure $P_{\mathrm{ext}} = P_2 = 1\,\mathrm{atm}$:

$$\delta W = -P\,dV \Rightarrow W' = -P_2\int_{V_1}^{V_2} dV = -P_2(V_2 - V_1)$$

Given:

$$P_1V_1 = P_2V_2 = RT \Rightarrow V_1 = \frac{RT}{P_1}, \quad V_2 = \frac{RT}{P_2}$$

$$W' = RT\left(\frac{P_2}{P_1} - 1\right)$$

Numerical application: $W' = -1982\,\mathrm{J}$.

0.0.45 Exercise 23

A gas obeys the Van der Waals equation for one mole:

$$(P + \frac{a}{V^2})(V - b) = RT$$

where a and b are positive constants.

- In the International System of Units, what are the units of a and b?
- Write the Van der Waals equation for n moles.

0.0.46 Solution to Exercice 23

The units of constants a and b in the SI system are determined as follows:

$$[V - b] = [V] - [b] = [V] = [b] \implies [b] = L^3,$$

so the unit of b is m^3.

$$\left[P + \frac{a}{V^2}\right] = [P] + \left[\frac{a}{V^2}\right] = [P] = \left[\frac{a}{V^2}\right] \implies [a] = [P][V^2],$$

where

$$[P] = \frac{[F]}{[S]} = \frac{([m] \cdot [X] \cdot [t^{-2}])}{([X] \cdot [X])} = \frac{([m] \cdot [t^{-2}])}{[X]} = ML^{-1}T^{-2},$$

and

$$[V^2] = [X^3]^2 = L^6.$$

Thus,

$$[a] = ML^{-1}T^{-2}L^6 = ML^5T^{-2},$$

so the unit of a is $\mathrm{kg} \cdot \mathrm{m}^5 \cdot \mathrm{s}^{-2}$.

Van der Waals Equation for n Moles

For one mole, the equation of state as a function of P_0, V_0, and T_0 is:

$$\left(P_0 + \frac{a}{V_0^2}\right)(V_0 - b) = RT_0.$$

For n moles, the equation of state as a function of P, V, and T is derived. Here:

$$\begin{cases} V = nV_0 & \text{(extensive)}, \\ P = P_0 & \text{(intensive)}, \\ T = T_0 & \text{(intensive)}. \end{cases}$$

Substituting these relationships into the equation:

$$\left(P_0 + \frac{a}{V_0^2}\right)(V_0 - b) = RT_0 \iff \left(P + \frac{a}{(V/n)^2}\right)\left(\frac{V}{n} - b\right) = RT.$$

Simplifying further:

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT.$$

Thus, the equation of state for n moles is:

$$\boxed{\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT.}$$

0.0.47 Exercise 24

A piece of ice with mass $M = 500\,\mathrm{g}$, initially at $-10°\mathrm{C}$, receives heat at a rate of $q = 175\,\mathrm{J/s}$, according to the following diagram:

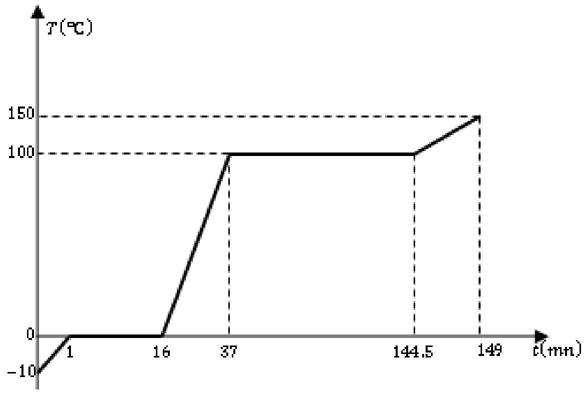


Figure 10: The diagram T(t).

- Describe the different phases.
- Evaluate the latent heats of phase change.
- Evaluate the specific heats of each phase and the molar heat of water vapor.

0.0.48 Solution to Exercise 24

Different Phases:

- Phase 1 ($t \in [0, 1\,\mathrm{min}]$): The ice warms up from $-10°\mathrm{C}$ to $0°\mathrm{C}$.
- Phase 2 ($t \in [1, 16\,\mathrm{min}]$): The ice melts at $0°\mathrm{C}$ and turns into liquid.
- Phase 3 ($t \in [16, 37\,\mathrm{min}]$): The liquid from phase 2 heats up from $0°\mathrm{C}$ to $100°\mathrm{C}$.
- Phase 4 ($t \in [37, 144.5\,\mathrm{min}]$): The liquid evaporates at a constant temperature of $100°\mathrm{C}$.
- Phase 5 ($t \in [144.5, 149\,\mathrm{min}]$): The vapor from the previous phase heats up from $100°\mathrm{C}$ to $150°\mathrm{C}$.

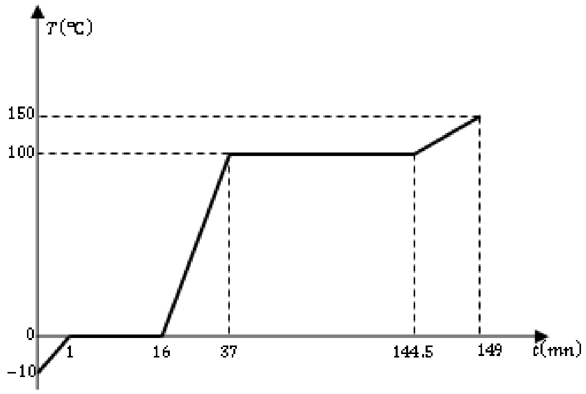


Figure 11: The diagram T(t).

Latent Heats of Phase Change:

The latent heat is defined as:

$$Q = m \cdot L = q \cdot \Delta t \Rightarrow L = \frac{q \cdot \Delta t}{m}$$

- Phase 2 ($t \in [1, 16\,\text{min}]$): $L_F = 336\,\text{J/g}$.
- Phase 4 ($t \in [37, 144.5\,\text{min}]$): $L_F = 2257.5\,\text{J/g}$.

Specific Heats of Each Phase:

The specific heat is defined as:

$$Q = m \cdot c_P \cdot \Delta T = q \cdot \Delta t \Rightarrow c_P = \frac{q \cdot \Delta t}{m \cdot \Delta T}$$

where c_P is the specific heat at constant pressure.

- Phase 1 ($t \in [0, 1\,\text{min}]$): $c_P = 21\,\text{J/g} \cdot \text{K}$.
- Phase 3 ($t \in [16, 37\,\text{min}]$): $c_P = 4.41\,\text{J/g} \cdot \text{K}$.
- Phase 5 ($t \in [144.5, 149\,\text{min}]$): $c_P = 1.89\,\text{J/g} \cdot \text{K}$.

The molar heat of water vapor is:

$$c_P = \frac{q \cdot \Delta t}{m \cdot \Delta T} = \frac{q \cdot \Delta t}{n \cdot M \cdot \Delta T} \Rightarrow M \cdot c_P = \frac{q \cdot \Delta t}{n \cdot \Delta T}$$

where M is the molar mass of water.

$$C_P = (2M_H + M_O)c_P = (2 + 16) \cdot 1.89 = 34.02\,\text{J/mol} \cdot \text{K}.$$

0.0.49 Exercise 25

In a calorimeter with negligible heat capacity containing $m_1 = 200\,\text{g}$ of water at $T_1 = 50°\text{C}$, a mass $m_2 = 200\,\text{g}$ at $T_2 = -10°\text{C}$ is dropped. The mixture reaches an equilibrium state at temperature T_f.

- Determine the final temperature T_f.

0.0.50 Solution to Exercice 25

Step 1: Heat Exchange Analysis The system involves three key processes:

- 1. The cold mass m_2 warms up from $T_2 = -10°\text{C}$ to T_f.
- 2. Some or all of the cold mass may melt if it is ice (latent heat of fusion).
- 3. The warm water m_1 cools down from $T_1 = 50°\text{C}$ to T_f.

Since $T_f > T_0$, we assume that all the ice melts completely during the process.

Step 2: Heat Balance Equation The principle of conservation of energy states that the total heat lost by the warm water equals the total heat gained by the cold mass (including both warming and melting). Mathematically:

$$Q_{\text{lost}} = Q_{\text{gained}},$$

where:

- - Q_{lost} is the heat lost by the warm water,
- - Q_{gained} includes the heat required to warm the cold mass and melt it.

The heat balance equation becomes:

$$m_1 c_w (T_1 - T_f) = m_2 c_i (T_0 - T_2) + m_2 L_f + m_2 c_w (T_f - T_0),$$

where:

- - $c_w = 4.18\,\text{J/g}^{\circ}\text{C}$ is the specific heat of water,
- - $c_i = 2.09\,\text{J/g}^{\circ}\text{C}$ is the specific heat of ice,
- - $L_f = 334\,\text{J/g}$ is the latent heat of fusion of ice.

Step 3: Substitute Known Values Substitute the given values:

- - $m_1 = 200\,\text{g}$,
- - $m_2 = 200\,\text{g}$,
- - $T_1 = 50°\text{C}$,
- - $T_2 = -10°\text{C}$,
- - $T_0 = 0°\text{C}$,
- - $c_w = 4.18\,\text{J/g}^{\circ}\text{C}$,
- - $c_i = 2.09\,\text{J/g}^{\circ}\text{C}$,
- - $L_f = 334\,\text{J/g}$.

The equation becomes:

$$200 \cdot 4.18 \cdot (50 - T_f) = 200 \cdot 2.09 \cdot (0 - (-10)) + 200 \cdot 334 + 200 \cdot 4.18 \cdot (T_f - 0).$$

Simplify each term:

- - Left-hand side: $200 \cdot 4.18 \cdot (50 - T_f) = 836 \cdot (50 - T_f) = 41800 - 836T_f$,
- - First term on the right-hand side: $200 \cdot 2.09 \cdot 10 = 4180$,
- - Second term on the right-hand side: $200 \cdot 334 = 66800$,
- - Third term on the right-hand side: $200 \cdot 4.18 \cdot T_f = 836T_f$.

Thus, the equation becomes:

$$41800 - 836T_f = 4180 + 66800 + 836T_f.$$

Step 4: Solve for T_f Combine like terms:

$$41800 - 4180 - 66800 = 836T_f + 836T_f.$$

Simplify:

$$-29180 = 1672T_f.$$

Solve for T_f:

$$T_f = \frac{-29180}{1672} \approx 17.45°\text{C}.$$

Final Answer:

$$\boxed{T_f \approx 17.45°\text{C}}$$

0.0.51 Exercise 26

Let f be a state function defined by the three state variables P, V, and T of a system such that $f(P, V, T) = 0$.

- Write the differentials dV, dP, and dT and deduce the following Reech formulas:

$$\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial V}\right)_P = 1$$
$$\left(\frac{\partial V}{\partial P}\right)_T \left(\frac{\partial P}{\partial V}\right)_T = 1$$
$$\left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial T}\right)_V = 1$$
$$\left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = -1$$

0.0.52 Solution to Exercice 26

The differentials are given as:

$$V(p,T) \xrightarrow{\text{differential}} dV = \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT \quad \ldots(1)$$

$$p(V,T) \xrightarrow{\text{differential}} dp = \left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V dT \quad \ldots(2)$$

$$T(p,V) \xrightarrow{\text{differential}} dT = \left(\frac{\partial T}{\partial p}\right)_V dp + \left(\frac{\partial T}{\partial V}\right)_p dV \quad \ldots(3)$$

Case 1: Substituting (3) into (1) and (2)

(3) into (1):

Substituting dT from (3) into (1):

$$dV = \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT = \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p \left[\left(\frac{\partial T}{\partial p}\right)_V dp + \left(\frac{\partial T}{\partial V}\right)_p dV\right]$$

Expanding:

$$dV = \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V dp + \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial V}\right)_p dV$$

$$dV = \left[\left(\frac{\partial V}{\partial p}\right)_T + \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V\right] dp + \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial V}\right)_p dV$$

By identification, we obtain:

$$\begin{cases} \left(\frac{\partial V}{\partial p}\right)_T + \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = 0 \\ \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial V}\right)_p = 1 \quad \Longrightarrow \quad \left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{\left(\frac{\partial T}{\partial V}\right)_p} \end{cases} \qquad \dots(4)$$

(3) into (2):

Substituting dT from (3) into (2):

$$dp = \left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V dT = \left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V \left[\left(\frac{\partial T}{\partial p}\right)_V dp + \left(\frac{\partial T}{\partial V}\right)_p dV\right]$$

Expanding:

$$dp = \left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial p}\right)_V dp + \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_p dV$$

$$dp = \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial p}\right)_V dp + \left[\left(\frac{\partial p}{\partial V}\right)_T + \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_p\right] dV$$

By identification, we obtain:

$$\begin{cases} \left(\frac{\partial p}{\partial V}\right)_T + \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_p = 0 \\ \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial p}\right)_V = 1 \quad \Longrightarrow \quad \left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{\left(\frac{\partial T}{\partial p}\right)_V} \end{cases} \qquad \dots(5)$$

From (4) and (5), we have:

$$\left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -\left(\frac{\partial V}{\partial p}\right)_T \quad \Longrightarrow \quad \frac{1}{\left(\frac{\partial T}{\partial V}\right)_p} \cdot \frac{1}{\left(\frac{\partial p}{\partial T}\right)_V} = -\left(\frac{\partial V}{\partial p}\right)_T$$

$$\left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_p = -1 \quad \dots(6)$$

Case 2: Substituting (2) into (1) and (3)

(2) into (1):

Substituting dp from (2) into (1):

$$dV = \left(\frac{\partial V}{\partial p}\right)_T \left[\left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V dT\right] + \left(\frac{\partial V}{\partial T}\right)_p dT$$

Expanding:

$$dV = \left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T dV + \left[\left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_V + \left(\frac{\partial V}{\partial T}\right)_p\right] dT$$

By identification, we obtain:

$$\begin{cases} \left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_V + \left(\frac{\partial V}{\partial T}\right)_p = 0 \\ \left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T = 1 \quad \Longrightarrow \quad \left(\frac{\partial V}{\partial p}\right)_T = \frac{1}{\left(\frac{\partial p}{\partial V}\right)_T} \end{cases} \quad \ldots (7)$$

(2) into (3):

Substituting dp from (2) into (3):

$$dT = \left(\frac{\partial T}{\partial p}\right)_V \left[\left(\frac{\partial p}{\partial V}\right)_T dV + \left(\frac{\partial p}{\partial T}\right)_V dT\right] + \left(\frac{\partial T}{\partial V}\right)_p dV$$

Expanding:

$$dT = \left(\frac{\partial T}{\partial p}\right)_V \left(\frac{\partial p}{\partial T}\right)_V dT + \left[\left(\frac{\partial T}{\partial p}\right)_V \left(\frac{\partial p}{\partial V}\right)_T + \left(\frac{\partial T}{\partial V}\right)_p\right] dV$$

By identification, we obtain:

$$\begin{cases} \left(\frac{\partial T}{\partial p}\right)_V \left(\frac{\partial p}{\partial V}\right)_T + \left(\frac{\partial T}{\partial V}\right)_p = 0 \\ \left(\frac{\partial T}{\partial p}\right)_V \left(\frac{\partial p}{\partial T}\right)_V = 1 \quad \Longrightarrow \quad \left(\frac{\partial T}{\partial p}\right)_V = \frac{1}{\left(\frac{\partial p}{\partial T}\right)_V} \end{cases} \quad \ldots (8)$$

From (7) and (8), we have:

$$\left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_V = -\left(\frac{\partial V}{\partial T}\right)_p \quad \Longrightarrow \quad \frac{1}{\left(\frac{\partial p}{\partial V}\right)_T} \cdot \frac{1}{\left(\frac{\partial T}{\partial p}\right)_V} = -\left(\frac{\partial V}{\partial T}\right)_p$$

$$\left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial T}{\partial p}\right)_V = -1 \quad \ldots (9)$$

Case 3: Substituting (1) into (2) and (3)

(1) into (2):

Substituting dV from (1) into (2):

$$dp = \left(\frac{\partial p}{\partial V}\right)_T \left[\left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT\right] + \left(\frac{\partial p}{\partial T}\right)_V dT$$

Expanding:

$$dp = \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial p}\right)_T dp + \left[\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p + \left(\frac{\partial p}{\partial T}\right)_V\right] dT$$

By identification, we obtain:

$$\begin{cases} \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p + \left(\frac{\partial p}{\partial T}\right)_V = 0 \\ \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial p}\right)_T = 1 \quad \Longrightarrow \quad \left(\frac{\partial p}{\partial V}\right)_T = \frac{1}{\left(\frac{\partial V}{\partial p}\right)_T} \end{cases} \quad \ldots (10)$$

(1) into (3):

Substituting dV from (1) into (3):

$$dT = \left(\frac{\partial T}{\partial p}\right)_V dp + \left(\frac{\partial T}{\partial V}\right)_p \left[\left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT\right]$$

Expanding:

$$dT = \left(\frac{\partial T}{\partial V}\right)_p \left(\frac{\partial V}{\partial T}\right)_p dT + \left[\left(\frac{\partial T}{\partial p}\right)_V + \left(\frac{\partial T}{\partial V}\right)_p \left(\frac{\partial V}{\partial p}\right)_T\right] dp$$

By identification, we obtain:

$$\begin{cases} \left(\frac{\partial T}{\partial p}\right)_V + \left(\frac{\partial T}{\partial V}\right)_p \left(\frac{\partial V}{\partial p}\right)_T = 0 \\ \left(\frac{\partial T}{\partial V}\right)_p \left(\frac{\partial V}{\partial T}\right)_p = 1 \quad \Longrightarrow \quad \left(\frac{\partial T}{\partial V}\right)_p = \frac{1}{\left(\frac{\partial V}{\partial T}\right)_p} \end{cases} \quad \ldots (11)$$

From (10) and (11), we have:

$$\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p + \left(\frac{\partial p}{\partial T}\right)_V = 0 \quad \Longrightarrow \quad \frac{1}{\left(\frac{\partial V}{\partial p}\right)_T} \cdot \frac{1}{\left(\frac{\partial T}{\partial V}\right)_p} = -\left(\frac{\partial p}{\partial T}\right)_V$$

$$\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial T}{\partial V}\right)_p = -1 \quad \ldots (12)$$

0.0.53 Exercise 27

A container holds 10 liters of water at $T_0 = 60°\text{C}$.

- What volume of water at $T_0 = 15°\text{C}$ must be added to obtain water at $T_0 = 30°\text{C}$?
- In the container with 10 liters of water at 60°C, a piece of ice with mass $M = 800\,\text{g}$ at $T_g = 0°\text{C}$ is added. What is the final temperature of the mixture?
- In the container with 10 liters of water at 60°C, a resistor $R = 600\,\Omega$ carrying a current $I = 1.2\,\text{A}$ is immersed.

- How long will it take to bring the water to boiling?
- Once boiling is reached, how long will it take to vaporize 0.5 liters of water?

Assume negligible losses and constant pressure of 1 atm. Given: $C_{Pe} = 4.18\,\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$, $L_V = 2260\,\text{kJ} \cdot \text{kg}^{-1}$, $L_F = 334\,\text{kJ} \cdot \text{kg}^{-1}$, $C_{Pg} = 2.09\,\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$, $\rho_e = 1\,\text{kg} \cdot \text{m}^{-3}$, $\rho_g = 917\,\text{kg} \cdot \text{m}^{-3}$.

0.0.54 Solution to Exercise 27

1. Mixing Water at Different Temperatures

Given:

- Initial volume of hot water, $V_1 = 10\,\text{L}$ at $T_1 = 60°\text{C}$.
- Cold water temperature, $T_2 = 15°\text{C}$.
- Desired final temperature, $T_f = 30°\text{C}$.
- Density of water, $\rho_e = 1\,\text{kg/L}$.
- Specific heat capacity of water, $C_{Pe} = 4.18\,\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$.

Volume of cold water (V_2) to add.
The heat lost by the hot water equals the heat gained by the cold water:

$$m_1 C_{Pe}(T_1 - T_f) = m_2 C_{Pe}(T_f - T_2)$$

Since $m = \rho V$, and ρ cancels out:

$$10\,\text{kg} \cdot (60 - 30) = V_2 \cdot (30 - 15)$$

$$300 = 15 V_2$$

$$V_2 = \frac{300}{15} = 20\,\text{L}$$

$\boxed{V_2 = 20\,\mathbf{L}}$

2. Adding Ice to Hot Water

Given:

- Mass of ice, $m_g = 800\,\text{g} = 0.8\,\text{kg}$ at $T_g = 0°\text{C}$.
- Latent heat of fusion, $L_F = 334\,\text{kJ/kg}$.
- Specific heat capacity of ice, $C_{Pg} = 2.09\,\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$.
- Initial water mass, $m_e = 10\,\text{kg}$ at 60°C.

Final temperature (T_f) of the mixture.
First, check if all the ice melts:

$$\text{Heat required to melt ice: } Q_{\text{melt}} = m_g L_F = 0.8 \times 334 = 267.2\,\text{kJ}$$

Heat released by water cooling to 0°C: $Q_{\text{water}} = m_e C_{Pe}(60-0) = 10\times4.18\times60 = 2508\,\text{kJ}$

Since $Q_{\text{water}} > Q_{\text{melt}}$, all ice melts. Now, find T_f:

$$\text{Total heat absorbed by ice: } Q_{\text{ice}} = m_g L_F + m_g C_{Pe}(T_f - 0)$$

$$\text{Heat lost by water: } Q_{\text{water}} = m_e C_{Pe}(60 - T_f)$$

Setting $Q_{\text{water}} = Q_{\text{ice}}$:

$$10 \times 4.18 \times (60 - T_f) = 0.8 \times 334 + 0.8 \times 4.18 \times T_f$$

$$2508 - 41.8T_f = 267.2 + 3.344T_f$$

$$2508 - 267.2 = 41.8T_f + 3.344T_f$$

$$2240.8 = 45.144T_f$$

$$T_f = \frac{2240.8}{45.144} \approx 49.6°\text{C}$$

$\boxed{T_f = 49.6°\mathbf{C}}$

3. Heating Water with a Resistor

Given:

- Resistance, $R = 600\,\Omega$.
- Current, $I = 1.2\,\text{A}$.
- Initial water temperature, $T_0 = 60°\text{C}$.
- Boiling point, $T_b = 100°\text{C}$.
- Mass of water, $m = 10\,\text{kg}$.
- Specific heat capacity, $C_{Pe} = 4.18\,\text{kJ}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$.

Time (t) to reach boiling.
Power dissipated by the resistor:

$$P = I^2 R = (1.2)^2 \times 600 = 864\,\text{W} = 0.864\,\text{kW}$$

Heat required to boil the water:

$$Q = mC_{Pe}(T_b - T_0) = 10 \times 4.18 \times (100 - 60) = 1672\,\text{kJ}$$

Time required:

$$t = \frac{Q}{P} = \frac{1672}{0.864} \approx 1935\,\text{s} \approx 32.25\,\text{min}$$

Answer: $\boxed{t = 1935\,\text{s}}$

4. Vaporizing Water

Given:

- Volume to vaporize, $V = 0.5\,\mathrm{L}$.
- Mass, $m = 0.5\,\mathrm{kg}$ (since $\rho_e = 1\,\mathrm{kg/L}$).
- Latent heat of vaporization, $L_V = 2260\,\mathrm{kJ/kg}$.
- Power, $P = 0.864\,\mathrm{kW}$.

Time (t) to vaporize $0.5\,\mathrm{L}$.
Heat required for vaporization:

$$Q = mL_V = 0.5 \times 2260 = 1130\,\mathrm{kJ}$$

Time required:

$$t = \frac{Q}{P} = \frac{1130}{0.864} \approx 1308\,\mathrm{s} \approx 21.8\,\mathrm{min}$$

Answer: $\boxed{t = 1308\,\mathrm{s}}$

0.0.55 Exercise 28

A closed container with a movable piston contains 2 g of helium (an ideal monatomic gas) under conditions (P_1, V_1). A reversible adiabatic compression is performed, bringing the gas to conditions (P_2, V_2). Given: $P_1 = 1\,\mathrm{bar}$, $V_1 = 10\,\mathrm{l}$, $P_2 = 3\,\mathrm{bar}$.

- Determine the final volume V_2 of the gas.
- Calculate the work exchanged by the gas with the surroundings.
- Determine the change in internal energy of the gas.
- Deduce the change in temperature of the gas without calculating its initial temperature.

Given: $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$, $R = 8.32\,\mathrm{J \cdot K^{-1} \cdot mol^{-1}}$.

0.0.56 Solution to Exercise 28

Final Volume of the Gas V_2:

Adiabatic transformation $\Rightarrow pV^\gamma =$ constant.

$$p_1 V_1^\gamma = p_2 V_2^\gamma \Rightarrow V_2^\gamma = \frac{p_1}{p_2} V_1^\gamma$$

$$\Rightarrow V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} V_1$$

Numerical Application (N.A.): $V_2 = 5.17\,\mathrm{L}$.

Work Done by the Gas:

First, differentiate the expression $pV^\gamma = \text{constant}$:

$$V^\gamma dp + \gamma p V^{\gamma-1} dV = 0$$

The work is: $\delta W = -pdV$.

$$\Rightarrow W = -\int_{V_1}^{V_2} pdV = -\int_{V_1}^{V_2} \frac{\text{constant}}{V^\gamma} dV = -\text{constant} \int_{V_1}^{V_2} \frac{dV}{V^\gamma} = -\text{constant} \left. \frac{V^{1-\gamma}}{1-\gamma} \right|_{V_1}^{V_2}$$

$$= \frac{\text{constant}}{1-\gamma} \left(V_1^{1-\gamma} - V_2^{1-\gamma} \right) = \frac{1}{1-\gamma} \left(V_1 \frac{\text{constant}}{V_1^\gamma} - V_2 \frac{\text{constant}}{V_2^\gamma} \right)$$

$$\Rightarrow W = \frac{1}{1-\gamma} \left(V_1 p_1 - V_2 p_2 \right)$$

N.A.: $W = 826.5\,\text{J}$.

Change in Internal Energy of the Gas:

Adiabatic transformation $\Rightarrow Q = 0$.

$$\Delta U = W + Q = W = \frac{1}{1-\gamma} \left(V_1 p_1 - V_2 p_2 \right)$$

$$\Rightarrow \Delta U = \frac{1}{1-\gamma} \left(V_1 p_1 - V_2 p_2 \right)$$

N.A.: $\Delta U = 826.5\,\text{J}$.

Change in Temperature:

$$\Delta U = nC_V \Delta T \Rightarrow \Delta T = \frac{\Delta U}{nC_V}$$

With $n = \frac{m}{M}$, where m is the mass of the gas and M is the molar mass of the gas.

$$C_V = \frac{R}{\gamma - 1} \quad \text{(molar heat capacity at constant volume)}$$

$$\Rightarrow \Delta T = \frac{\frac{1}{1-\gamma} \left(V_1 p_1 - V_2 p_2 \right)}{\frac{m}{M} \frac{R}{\gamma-1}} = \frac{M \left(V_2 p_2 - V_1 p_1 \right)}{mR}$$

N.A.: $\Delta T = -198.67\,\text{K}$.

0.0.57 Exercise 29

Consider one mole of an ideal gas with a constant ratio $\gamma = \frac{C_p}{C_v}$. The gas undergoes a polytropic transformation from state 1 (P_1, V_1, T_1) to state 2 (P_2, V_2, T_2).

- Show that the work exchanged W is given by:

$$W = \frac{P_2V_2 - P_1V_1}{k-1}$$

- Show that the heat Q is given by:

$$Q = C_m(T_2 - T_1)$$

Express C_m in terms of k and γ.

0.0.58 Solution to Exercise 29

Work:

Polytropic transformation $pV^K =$ constant, assuming this constant is A.

$$\Rightarrow p = \frac{A}{V^K}$$

$$W = -\int_{V_1}^{V_2} pdV = -A\int_{V_1}^{V_2} \frac{dV}{V^K} = -A\left.\frac{V^{1-K}}{1-K}\right|_{V_1}^{V_2} = \frac{1}{1-K}\left(V_1\frac{A}{V_1^K} - V_2\frac{A}{V_2^K}\right)$$

$$\Rightarrow W = \frac{1}{1-K}(V_1p_1 - V_2p_2) = \frac{1}{K-1}(V_2p_2 - V_1p_1)$$

Heat:

$$\delta Q = dU - \delta W = nC_VdT + pdV \Rightarrow Q = \Delta U - W = nC_V(T_2 - T_1) - \frac{1}{K-1}(V_2p_2 - V_1p_1)$$

For one mole:

$$\begin{cases} V_1p_1 = RT_1 \\ V_2p_2 = RT_2 \end{cases}$$

Thus,

$$Q = nC_V(T_2 - T_1) - \frac{1}{K-1}(RT_2 - RT_1) = nC_V(T_2 - T_1) - \frac{R}{K-1}(T_2 - T_1)$$

$$\Rightarrow Q = (T_2 - T_1)\left(nC_V - \frac{R}{K-1}\right)$$

Thus, the heat can be written as: $Q = C_m(T_2 - T_1)$, where:

$$C_m = nC_V - \frac{R}{K-1}$$

With $C_V = \frac{R}{\gamma-1}$ and $n = 1$ (1 mole), we obtain:

$$\Rightarrow C_m = \frac{R}{\gamma-1} - \frac{R}{K-1} = R\frac{K-\gamma}{(\gamma-1)(K-1)}$$

0.0.59 Exercise 30

A compression from 1 bar to 10 bars is performed on one liter of air, assumed to be an ideal gas initially at room temperature 20°C. The compression is rapid enough that the container does not have time to dissipate heat during the compression ($dQ = 0$).

- Determine the final temperature T_2 (corresponding to the state at 10 bars).
- Determine the final volume V_2 of the gas.
- Determine the change in internal energy of the gas.

Given for air: $\gamma = 1.4$, $r = 287.1\,\mathrm{J\cdot K^{-1}\cdot kg^{-1}}$, $c_V = 0.55\,\mathrm{J\cdot K^{-1}\cdot kg^{-1}}$.

0.0.60 Solution to Exercise 30

Final Temperature T_2:

$$dQ = 0 \Leftrightarrow \text{Adiabatic transformation}$$

$$\Rightarrow pV^\gamma = \text{constant}$$

Since the gas is ideal, we can write: $pV = nRT \Rightarrow V = \frac{nRT}{p}$.
Thus,

$$\Rightarrow pV^\gamma = p\left(\frac{nRT}{p}\right)^\gamma = (nRT)^\gamma p^{1-\gamma} = \text{constant}$$

$$\Rightarrow T^\gamma p^{1-\gamma} = \text{constant}$$

Therefore, from the initial to the final state, we can write:

$$T_1^\gamma p_1^{1-\gamma} = T_2^\gamma p_2^{1-\gamma} \Rightarrow T_2^\gamma = T_1^\gamma \frac{p_1^{1-\gamma}}{p_2^{1-\gamma}}$$

$$\Rightarrow T_2 = T_1\left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}}$$

N.A.: $T_2 = 10.36\,°\mathrm{C} = 283.36\,\mathrm{K}$.

Final Volume of the Gas V_2:

$$p_1V_1^\gamma = p_2V_2^\gamma \Rightarrow V_2^\gamma = \frac{p_1}{p_2}V_1^\gamma$$

$$\Rightarrow V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} V_1$$

N.A.: $V_2 = 0.19\,\mathrm{L}$.

Change in Internal Energy of the Gas:

$$\Delta U = Q + W = 0 + W = W = mc_V\Delta T$$

Where m is the mass of the gas and c_V is the specific heat capacity at constant volume.

The ideal gas equation is: $pV = nRT = \frac{m}{M}RT = mrT$, where r is the specific gas constant.

$$\Rightarrow pV = mrT \Rightarrow p_1V_1 = mrT_1 \Rightarrow m = \frac{p_1V_1}{rT_1}$$

$$\Rightarrow \Delta U = mc_V\Delta T = \frac{p_1V_1}{rT_1}c_V(T_2 - T_1)$$

N.A.: $\Delta U = 0.2\,\mathrm{J}$.

0.0.61 Exercise 31

A mass $m = 5\,\mathrm{kg}$ of helium gas ($M_{He} = 4\,\mathrm{g}\cdot\mathrm{mol}^{-1}$) undergoes a reversible polytropic expansion in a closed system from an initial pressure $P_1 = 10\,\mathrm{bar}$ and temperature $T_1 = 400°\mathrm{C}$ to a final pressure $P_2 = 2\,\mathrm{bar}$ and temperature $T_2 = 120°\mathrm{C}$. The helium gas is assumed to be ideal. Calculate:

- The volumes V_1 and V_2.
- The coefficient k.
- The work exchanged by the gas W.
- The heat Q.

0.0.62 Solution to Exercise 31

Calculation of Volumes V_1 and V_2:

Ideal gas $\Rightarrow$

$$\begin{cases} p_1V_1 = nRT_1 \\ p_2V_2 = nRT_2 \end{cases} \Rightarrow \begin{cases} V_1 = \frac{nRT_1}{p_1} = \frac{m}{M}\frac{RT_1}{p_1} \\ V_2 = \frac{nRT_2}{p_2} = \frac{m}{M}\frac{RT_2}{p_2} \end{cases}$$

N.A.:

$$\begin{cases} V_1 = 7\,\mathrm{m}^3 \\ V_2 = 20.44\,\mathrm{m}^3 \end{cases}$$

Coefficient K:

Polytropic transformation $\Rightarrow pV^K =$ constant.

$$\Rightarrow p_1V_1^K = p_2V_2^K \Rightarrow \left(\frac{V_1}{V_2}\right)^K = \frac{p_2}{p_1} \Rightarrow \ln\left(\frac{V_1}{V_2}\right)^K = \ln\left(\frac{p_2}{p_1}\right) \Rightarrow K\ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{p_2}{p_1}\right)$$

$$\Rightarrow K = \frac{\ln\left(\frac{p_2}{p_1}\right)}{\ln\left(\frac{V_1}{V_2}\right)}$$

N.A.: $K = 1.5$.

Work Exchanged by the Gas:

$$W = -\int_{V_1}^{V_2} pdV$$

Polytropic transformation $\Rightarrow pV^K =$ constant $= A \Rightarrow p = \frac{A}{V^K}$. Substituting into the previous expression, we obtain:

$$W = -\int_{V_1}^{V_2} pdV = -A\int_{V_1}^{V_2} \frac{dV}{V^K} = A\left.\frac{V^{1-K}}{1-K}\right|_{V_1}^{V_2} = \frac{A}{1-K}\left(V_1^{1-K} - V_2^{1-K}\right)$$

$$= \frac{1}{1-K}\left(V_1\frac{A}{V_1^K} - V_2\frac{A}{V_2^K}\right)$$

$$\Rightarrow W = \frac{1}{1-K}\left(V_1p_1 - V_2p_2\right)$$

N.A.: $W = -5824 \times 10^3$ J. Work is transferred to the surroundings.

Quantity of Heat:

$$\Delta U = Q + W = mc_V\Delta T = mc_V(T_2 - T_1)$$

$$\Rightarrow Q = mc_V(T_2 - T_1) - W$$

N.A.: $Q = 5806528$ J.

0.0.63 Exercise 32

A cylinder contains one mole of an ideal monatomic gas that undergoes a reversible cycle consisting of three transformations: an isothermal transformation (1-2), an isobaric transformation (2-3), and an isochoric transformation (3-1). Given: $T_1 = 300\,\text{K}$, $P_1 = 1\,\text{bar}$, $P_2 = 5\,\text{bar}$.

- Is the cycle (1-2-3-1) a power cycle or a refrigeration cycle? Justify your answer.

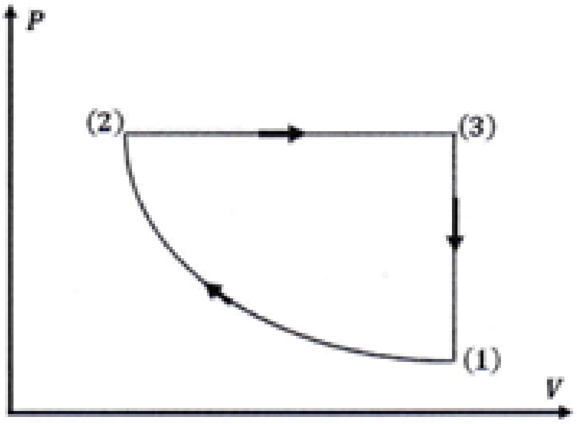


Figure 12: The cycle 1.

- Calculate:
  - The volumes V_1, V_2, and V_3.
  - The temperature T_3 of state (3).
  - The work W_{12}, W_{23}, and W_{31} involved in the cycle.
  - The heat Q_{12}, Q_{23}, and Q_{31} exchanged by the gas during the cycle.
  - The total work W and total heat Q received during the cycle. Is the first principle, applied to the cycle (1-2-3-1), verified?
- Deduce the efficiency of the cycle.

Given: $C_V = \frac{3}{2}nR$, $R = 8.32\,\mathrm{J\cdot K^{-1}\cdot mol^{-1}}$, $n = 1\,\mathrm{mol}$.

0.0.64 Solution to Exercise 32

Type of Cycle:

The cycle (1-2-3-1) is clockwise, so it is a power cycle.

Volumes:

For state (1), the volume is V_1. For one mole of an ideal gas, we have:

$$p_1V_1 = RT_1 \implies V_1 = \frac{RT_1}{p_1}$$

Numerical Application (N.A.): $V_1 = 0.0249\,\mathrm{m}^3$.

For state (2), the volume is V_2 and the pressure is p_2. The transformation (1-2) is isothermal ($T_1 = \text{constant}$):

$$p_1V_1 = RT_1 \quad \text{and} \quad p_2V_2 = RT_1 \implies p_1V_1 = p_2V_2 \implies V_2 = V_1\frac{p_1}{p_2}$$

N.A.: $V_2 = 0.0048\,\mathrm{m}^3$.

For state (3), the volume is V_1 and the pressure is p_2. The transformation (3-1) is isochoric ($V_1 = V_3 = 0.0249\,\mathrm{m}^3$).

Temperature of State (3):

State (3): $(V_1, p_2, T_3) \implies p_2 V_1 = RT_3 \implies T_3 = \frac{p_2 V_1}{R}$. N.A.: $T_3 = 1497\,\mathrm{K}$.

Work:

$$\delta W = -p\,dV$$

(1-2) Isothermal transformation ($T = T_1 = \text{constant}$):

$$W_{12} = -\int_1^2 p\,dV = -RT\int_1^2 \frac{dV}{V} = -RT\ln V\Big|_{V_2}^{V_1} \implies W_{12} = RT\ln\left(\frac{V_1}{V_2}\right)$$

N.A.: $W_{12} = 4143\,\mathrm{J}$.

(2-3) Isobaric transformation ($p = p_2 = \text{constant}$):

$$W_{23} = -\int_2^3 p\,dV = -p_2\int_2^3 dV = p_2(V_2 - V_1) \implies W_{23} = p_2(V_2 - V_1)$$

N.A.: $W_{23} = -10050\,\mathrm{J}$.

(3-1) Isochoric transformation ($V = \text{constant} \implies dV = 0$):

$$W_{31} = 0.$$

Heat:

For an ideal gas, $dU = nc_V dT$ (First Law of Joule).

(1-2) Isothermal transformation ($dT = 0$):

$$\Delta U = 0 = Q_{12} + W_{12} \implies Q_{12} = -W_{12} = -4143\,\mathrm{J}.$$

(2-3) Isobaric transformation:

$$dU = c_V dT \implies \Delta U = c_V(T_3 - T_2) = Q_{23} + W_{23} \implies Q_{23} = c_V(T_3 - T_2) - W_{23}.$$

N.A.: $Q_{23} = 24978\,\mathrm{J}$.

(3-1) Isochoric transformation:

$$\Delta U = c_V(T_1 - T_3) = Q_{31} + W_{31} = Q_{31} \implies Q_{31} = c_V(T_1 - T_3).$$

N.A.: $Q_{31} = -14928\,\mathrm{J}$.

Total Work and Total Heat:

$$W = W_{12} + W_{23} + W_{31} = 4143 - 10050 + 0 = -5907\,\mathrm{J}.$$

$$Q = Q_{12} + Q_{23} + Q_{31} = -4143 + 24978 - 14928 = +5907\,\mathrm{J}.$$

For a closed cycle, the change in internal energy is zero ($\Delta U = W + Q = 0$), and the First Law of Thermodynamics is verified.

Efficiency of the Cycle:

$$\rho = -\frac{W}{Q_{\text{received}}} = 0.236 \implies \rho \approx 24\%.$$

0.0.65 Exercise 33

One mole of an ideal gas undergoes the cycle represented in the figure below, consisting of two isobaric and two isochoric processes. Let $P_2 > P_1$ and $V_2 > V_1$.

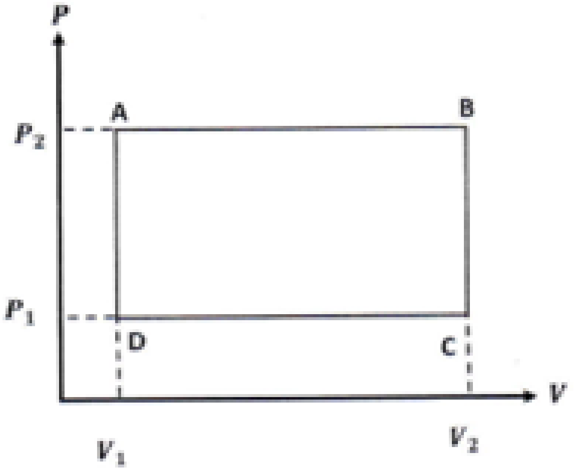


Figure 13: The cycle 2.

- In which direction should the cycle be traversed to act as a power cycle?
- Calculate the work involved in one cycle as a function of the pressures and volumes.
- Determine the temperatures at the vertices of the cycle if $P_2 = 2P_1 = 2\,\text{atm}$ and $V_2 = 2V_1 = 20\,\text{l}$.
- Calculate the efficiency of a Carnot engine operating between temperatures T_B and T_D.

Given: $R = 8.32\,\text{J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$, $1\,\text{atm} = 10^5\,\text{Pa}$.

0.0.66 Solution to Exercise 33

To Have an Engine:

The cycle must be traversed clockwise.

Total Work:

$$W_{12} = 0 \quad \text{and} \quad W_{34} = 0 \quad \text{(isochoric transformations).}$$

$$W_{23} = -p_2 \int_2^3 dV = -p_2(V_2 - V_1) = p_2(V_1 - V_2) \implies W_{23} = p_2(V_1 - V_2).$$

$$W_{41} = -p_2 \int_4^1 dV = -p_1(V_1 - V_2) = p_1(V_2 - V_1) \implies W_{41} = p_1(V_2 - V_1).$$

$$W = W_{12} + W_{23} + W_{34} + W_{41} = p_2(V_1 - V_2) + p_1(V_2 - V_1) = (V_2 - V_1)(p_1 - p_2).$$

Temperatures if $p_2 = 2p_1 = 2\,\mathbf{atm}$ and $V_2 = 2V_1 = 20\,\mathbf{L}$:

Vertex (1): $p_1V_1 = RT_1 \implies T_1 = \frac{p_1V_1}{R}$. N.A.: $T_1 = T_D = 120\,\mathrm{K}$.

Vertex (2): $p_2V_1 = RT_2 \implies T_2 = \frac{p_2V_1}{R}$. N.A.: $T_2 = T_A = 240.4\,\mathrm{K}$.

Vertex (3): $p_2V_2 = RT_3 \implies T_3 = \frac{p_2V_2}{R}$. N.A.: $T_3 = T_B = 480.8\,\mathrm{K}$.

Vertex (4): $p_1V_2 = RT_4 \implies T_4 = \frac{p_1V_2}{R}$. N.A.: $T_4 = T_C = 240.4\,\mathrm{K}$.

Efficiency of the Machine Between T_B and T_D:

$$\rho = -\frac{W_{\text{provided}}}{Q_{\text{received}}} = -\frac{-(Q_1 + Q_2)}{Q_2} = \frac{Q_1}{Q_2} + 1.$$

According to the Second Law of Thermodynamics: $S_1 + S_2 = 0 \iff \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$.

$$\implies \frac{Q_1}{Q_2} = -\frac{T_1}{T_2} \implies \rho = \frac{Q_1}{Q_2} + 1 = 1 - \frac{T_1}{T_2}.$$

N.A.: $\rho = 0.75 = 75\%$.

0.0.67 Exercise 34

Two isolated systems, labeled 1 and 2, initially at temperatures T_1 and T_2, with heat capacities at constant pressure C_{P1} and C_{P2}, are brought into thermal contact, with the pressure remaining constant during the process. After a sufficiently long time, an equilibrium state is reached where both systems have the same temperature T_f.

- Is the transformation reversible? Justify your answer.
- Calculate the heat Q_1 and Q_2 exchanged by each system, and show that the final temperature is:
$$T_f = \frac{C_{P1}T_1 + C_{P2}T_2}{C_{P1} + C_{P2}}$$
- Determine the expressions for the entropy changes ΔS_1 and ΔS_2 of each system and ΔS of the combined system.
- In the case $T_1 > T_2$, what are the signs of ΔS_1, ΔS_2, and ΔS?
- Determine the expression for ΔS in the case $C_{P1} = C_{P2} = C_P$, as a function of C_P, T_1, and T_2.

0.0.68 Solution to Exercise 34

The Transformation is Irreversible:

The transformation is irreversible because it is not possible to return to the initial state by the same path.

Heat Quantities Q_1 and Q_2:

$$Q_1 = c_{p_1}\Delta T = c_{p_1}(T_f - T_1) \quad \text{(heat quantity of system 1).}$$

$$Q_2 = c_{p_2}\Delta T = c_{p_2}(T_f - T_2) \quad \text{(heat quantity of system 2).}$$

At equilibrium: $\sum Q = 0 \implies Q_1 + Q_2 = 0 \implies c_{p_1}(T_f - T_1) + c_{p_2}(T_f - T_2) = 0 \implies T_f(c_{p_1} + c_{p_2}) = c_{p_1}T_1 + c_{p_2}T_2$.

$$\implies T_f = \frac{c_{p_1}T_1 + c_{p_2}T_2}{c_{p_1} + c_{p_2}}.$$

Expressions for Entropy Changes ΔS_1 and ΔS_2:

$$dS = \frac{\delta Q}{T} \implies \Delta S_1 = c_{p_1}\int_{T_1}^{T_f} \frac{\delta T}{T} = c_{p_1}\ln T\bigg|_{T_f}^{T_1} = c_{p_1}\ln\left(\frac{T_f}{T_1}\right).$$

$$\Delta S_2 = c_{p_2}\int_{T_2}^{T_f} \frac{\delta T}{T} = c_{p_2}\ln T\bigg|_{T_f}^{T_2} = c_{p_2}\ln\left(\frac{T_f}{T_2}\right).$$

Thus, the total entropy is:

$$\Delta S = \Delta S_1 + \Delta S_2 = c_{p_1}\ln\left(\frac{T_f}{T_1}\right) + c_{p_2}\ln\left(\frac{T_f}{T_2}\right).$$

Case $T_1 > T_2$: Signs of ΔS_1, ΔS_2, and ΔS:

If $T_1 > T_f > T_2$, then:

$$\ln\left(\frac{T_f}{T_1}\right) < 0 \implies \Delta S_1 < 0, \quad \ln\left(\frac{T_f}{T_2}\right) > 0 \implies \Delta S_2 > 0.$$

Thus, ΔS_1 is negative and ΔS_2 is positive.

$$\ln\left(\frac{T_f}{T_1}\right) < 0 < \ln\left(\frac{T_f}{T_2}\right) \implies c_{p_1}\ln\left(\frac{T_f}{T_1}\right) < 0 < c_{p_2}\ln\left(\frac{T_f}{T_2}\right) \implies \Delta S > 0.$$

Thus, ΔS is positive.

Expression for ΔS if $c_{p_1} = c_{p_2} = c_p$:

$$\Delta S = c_{p_1}\ln\left(\frac{T_f}{T_1}\right) + c_{p_2}\ln\left(\frac{T_f}{T_2}\right) = c_p\left[\ln\left(\frac{T_f}{T_1}\right) + \ln\left(\frac{T_f}{T_2}\right)\right] = c_p\ln\left[\left(\frac{T_f}{T_1}\right)\left(\frac{T_f}{T_2}\right)\right].$$

With $T_f = \frac{T_1+T_2}{2}$, we have:

$$\Delta S = c_p\ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right].$$

0.0.69 Exercise 35

A block of copper with mass $m = 2\,\text{kg}$ and temperature $T_c = 50°\text{C}$ is placed in an infinite medium at temperature $T_m = 25°\text{C}$. Under these conditions, the copper block cools from T_c to T_m. The specific heat of copper is $c = 385\,\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$.

- Calculate the heat Q exchanged by the copper block.
- Calculate the entropy change ΔS of this cooling process.
- Calculate the external entropy change ΔS_e.
- Calculate the created entropy ΔS_c.

0.0.70 Solution to Exercise 35

Heat Quantity Q Exchanged by the Block:

$$Q = mc\Delta T = mc(T_m - T_c).$$

N.A.: $Q = 2.385 \times (25 - 50) = -19250\,\text{J} = -19.25\,\text{kJ}$.

Entropy ΔS of Cooling:

$$dS = \frac{\delta Q}{T} = mc\frac{\delta T}{T} \implies \Delta S = mc\int_{T_c}^{T_m} \frac{dT}{T} = mc\ln\left(\frac{T_m}{T_c}\right).$$

N.A.: $\Delta S = -62.03\,\text{J} \cdot \text{K}^{-1}$.

Entropy S_e Exchanged with the Exterior:

$$\Delta S_e = \int \frac{\delta Q}{T_{\text{ext}}} = \frac{1}{T_m}\int \delta Q = \frac{Q}{T_m}.$$

N.A.: $\Delta S_e = \frac{-19250}{25+273} = -64.60\,\text{J} \cdot \text{K}^{-1}$.

Created Entropy ΔS_c:

$$\Delta S = \Delta S_e + \Delta S_c \implies \Delta S_c = \Delta S - \Delta S_e.$$

N.A.: $\Delta S_c = -62.03 + 64.60 = 2.57\,\text{J} \cdot \text{K}^{-1}$.

0.0.71 Exercise 36

During an infinitesimal reversible transformation of a thermodynamic system, the heat exchange δQ with the surroundings takes the following equivalent forms (for one mole):

$$\delta Q = C_V dT + l dV$$

$$\delta Q = C_P dT + h dP$$

- Differentiate dV and prove the relation:

$$C_P - C_V = l\left(\frac{\partial V}{\partial T}\right)_P$$

- Calculate the differentials dU and dS as functions of the independent variables T and V. Using the fact that dU and dS are exact differentials, show that:

$$l = T\left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$$

- What do these two relations become in the case of an ideal gas?

0.0.72 Solution to Exercise 36

The differential dV:

V varies as a function of p and T, so:

$$dV = \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT$$

We have:

$$\delta Q = C_V dT + l\, dV = C_V dT + l\left[\left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT\right]$$

$$\Rightarrow \delta Q = \left[C_V + l\left(\frac{\partial V}{\partial T}\right)_p\right] dT + l\left(\frac{\partial V}{\partial p}\right)_T dp$$

And:

$$\delta Q = C_p dT + h\, dp$$

By comparison, we find:

$$C_p = C_V + l\left(\frac{\partial V}{\partial T}\right)_p$$

$$\Rightarrow C_p - C_V = l\left(\frac{\partial V}{\partial T}\right)_p$$

The differentials dU and dS as functions of T and V:

First principle:

$$dU = \delta W + \delta Q = -p\, dV + C_V dT + l\, dV$$

$$\Rightarrow dU = C_V dT + (l - p)dV$$

Second principle:

$$dS = \frac{\delta Q}{T} = \frac{C_V}{T}dT + \frac{l}{T}dV$$

dU is an exact differential:

$$\left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial (l-p)}{\partial T}\right)_V$$

dS is an exact differential:

$$\left(\frac{\partial(C_V/T)}{\partial V}\right)_T = \left(\frac{\partial(l/T)}{\partial T}\right)_V$$

We obtain the following system of equations:

$$\begin{cases} \left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial l}{\partial T}\right)_V - \left(\frac{\partial p}{\partial T}\right)_V \\ \frac{1}{T}\left(\frac{\partial C_V}{\partial V}\right)_T = \frac{1}{T^2}\left[T\left(\frac{\partial l}{\partial T}\right)_V - l\right] \\ \Rightarrow \left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial l}{\partial T}\right)_V - \frac{l}{T} \end{cases}$$

$$\Rightarrow \left(\frac{\partial p}{\partial T}\right)_V = \frac{l}{T} \quad \Rightarrow \quad l = T\left(\frac{\partial p}{\partial T}\right)_V$$

Substituting into the first equation of the system, we obtain:

$$\left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial l}{\partial T}\right)_V - \left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial T}{\partial T}\right)_V \left(\frac{\partial p}{\partial T}\right)_V + T\left(\frac{\partial^2 p}{\partial T^2}\right)_V - \left(\frac{\partial p}{\partial T}\right)_V = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V$$

$$\Rightarrow \left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V$$

The case of an ideal gas:

$$pV = nRT \quad \Rightarrow \quad p = \frac{nRT}{V} \quad \Rightarrow \quad \left(\frac{\partial p}{\partial T}\right)_V = \frac{nR}{V} \quad \Rightarrow \quad \left(\frac{\partial^2 p}{\partial T^2}\right)_V = 0$$

$$\Rightarrow \begin{cases} l = T\left(\frac{\partial p}{\partial T}\right)_V = \frac{nRT}{V} = p \quad \Rightarrow \quad l = p \\ \left(\frac{\partial C_V}{\partial V}\right)_T = 0 \quad \Rightarrow \quad C_V = \text{constant} \end{cases}$$

0.0.73 Exercise 37

Consider an ideal gas undergoing an isobaric transformation from state (1) to state (2) in a closed system. The heat capacity at constant pressure C_P of the gas depends on temperature according to the equation:

$$C_P = \alpha T + \beta$$

where α and β are constants.

- Express the entropy change ΔS_{12} of this transformation as a function of α, β, T_1, and T_2.

0.0.74 Solution to Exercise 37

Isobaric transformation ($p =$ constant) of an ideal gas.

$$C_p = \alpha T + \beta \quad \text{(heat capacity at constant pressure).}$$

The entropy variation ΔS_{12} as a function of α, β, T_1, and T_2:

Second principle:

$$dS = \frac{\delta Q}{T} = \frac{1}{T}(C_V dT + l\,dV).$$

Where $C_p - C_V = nR$ (Mayer's relation).

$$\begin{cases} l = p \\ pV = nRT \end{cases} \quad \Rightarrow \quad \text{for an ideal gas.}$$

$$dS = C_V\frac{dT}{T} + \frac{p}{T}dV = (C_p - nR)\frac{dT}{T} + nR\frac{dV}{V}$$

$$\Rightarrow \Delta S_{12} = \int_1^2 C_p\frac{dT}{T} - nR\int_1^2\frac{dT}{T} + nR\int_1^2\frac{dV}{V} = \int_1^2(\alpha T+\beta)\frac{dT}{T} - nR\int_1^2\frac{dT}{T} + nR\int_1^2\frac{dV}{V}$$

$$= \alpha\int_1^2 dT + \beta\int_1^2\frac{dT}{T} - nR\int_1^2\frac{dT}{T} + nR\int_1^2\frac{dV}{V}$$

$$= \alpha(T_2 - T_1) + \beta\ln\left(\frac{T_2}{T_1}\right) - nR\ln\left(\frac{T_2}{T_1}\right) + nR\ln\left(\frac{V_2}{V_1}\right)$$

For an isobaric transformation:

$$\begin{cases} pV_1 = nRT_1 \\ pV_2 = nRT_2 \end{cases} \quad \Rightarrow \quad \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\Rightarrow \Delta S_{12} = \alpha(T_2 - T_1) + \beta\ln\left(\frac{T_2}{T_1}\right) - nR\ln\left(\frac{T_2}{T_1}\right) + nR\ln\left(\frac{T_2}{T_1}\right) = \alpha(T_2 - T_1) + \beta\ln\left(\frac{T_2}{T_1}\right)$$

$$\Rightarrow \Delta S_{12} = \alpha(T_2 - T_1) + \beta\ln\left(\frac{T_2}{T_1}\right)$$

0.0.75 Exercise 38

Consider one mole of carbon dioxide CO_2 with the Van der Waals equation of state:

$$(P + \frac{a}{V^2})(V - b) = RT$$

where a and b are positive constants.

- Given the heat exchanged by the gas during a reversible infinitesimal transformation:

  $$\delta Q = C_V dT + l dV$$

  where C_V is the molar specific heat at constant volume of CO_2 and $l = T\left(\frac{\partial P}{\partial T}\right)_V$. Calculate l.
- Calculate the infinitesimal change in internal energy dU.
- Calculate the change in internal energy ΔU of the carbon dioxide between the initial state $A(V_1, T_1)$ and the final state $B(V_2, T_2)$, assuming C_V is constant.

- Determine the temperature change $\Delta T = T_2 - T_1$ during an isoenergetic expansion ($U = \text{constant}$). Conclusion?

Given: $a = 0.366\,\text{J}\cdot\text{m}^3\cdot\text{mol}^{-2}$, $b = 4.29\times10^{-5}\,\text{m}^3\cdot\text{mol}^{-1}$, $V_1 = 0.2\times10^{-2}\,\text{m}^3\cdot\text{mol}^{-1}$, $V_2 = 3\times10^{-2}\,\text{m}^3\cdot\text{mol}^{-1}$, $T_1 = 293\,\text{K}$, $C_V = 28.5\,\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$.

0.0.76 Solution to Exercise 38

Calculation of l:

$$l = T\left(\frac{\partial p}{\partial T}\right)_V$$

We have:

$$\left(p + \frac{a}{V^2}\right)(V-b) = RT \quad\Rightarrow\quad p(V-b) + \frac{a}{V^2}(V-b) = RT \quad\Rightarrow\quad p = \frac{RT}{V-b} - \frac{a}{V^2}$$

$$\Rightarrow \left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{V-b} \quad\Rightarrow\quad l = \frac{RT}{V-b} = p + \frac{a}{V^2}$$

$$\Rightarrow l = p + \frac{a}{V^2}$$

The elementary variation dU:

$$dU = \delta Q + \delta W = C_V dT + l\,dV - p\,dV = C_V dT + (l-p)dV$$

$$\Rightarrow dU = C_V dT + \frac{a}{V^2}dV$$

The variation ΔU:

$$\Delta U = C_V\int_1^2 dT + a\int_1^2 \frac{dV}{V^2} = C_V(T_2 - T_1) - a\left.\frac{1}{V}\right|_{V_1}^{V_2}$$

$$\Rightarrow \Delta U = C_V(T_2 - T_1) - a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

The variation ΔT:

Isoenergetic expansion ($U = \text{constant}$) $\Rightarrow \Delta U = 0$.

$$\Rightarrow C_V(T_2 - T_1) - a\left(\frac{1}{V_2} - \frac{1}{V_1}\right) = 0 \quad\Rightarrow\quad (T_2 - T_1) = \frac{a}{C_V}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

$$\Rightarrow \Delta T = \frac{a}{C_V}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

Numerical application:

$$\Delta T = \frac{0.366}{28.5}\left(\frac{1}{0.03} - \frac{1}{0.002}\right) = -5.99 \approx -6\,\text{K} \quad\Rightarrow\quad \Delta T < 0.$$

Thus, the isoenergetic expansion cooled the CO_2.

0.0.77 Exercise 39

In an infinitesimal reversible transformation, the heat received by one mole of oxygen (an ideal gas) is given by:

$$\delta Q = C_P dT - V dP$$

- Write the infinitesimal entropy change dS of the oxygen.
- The molar specific heat of oxygen at constant pressure follows the law:

  $$C_P = a + bT + cT^2$$

  Calculate the final pressure of the oxygen compressed adiabatically and reversibly (isentropically) from the initial state ($T_0 = 273\,\mathrm{K}, P_0 = 1\,\mathrm{atm}$) to the temperature $T = 1000\,\mathrm{K}$.

Given: $R = 8.32\,\mathrm{J\cdot mol^{-1}\cdot K^{-1}}$, $a = 26.2\,\mathrm{J\cdot mol^{-1}\cdot K^{-1}}$, $b = 11.493\times10^{-3}\,\mathrm{J\cdot mol^{-1}\cdot K^{-2}}$, $c = -3.222\times10^{-6}\,\mathrm{J\cdot mol^{-1}\cdot K^{-3}}$.

0.0.78 Solution to Exercise 39

The elementary variation dS of O_2:

$$dS = \frac{\delta Q}{T} = C_p\frac{dT}{T} + \frac{h}{T}dp$$

Calculation of the final pressure:

Reversible adiabatic transformation ($\delta Q = 0$) ⇒ Isentropic transformation ($dS = 0$).

$$\Rightarrow C_p dT + h\,dp = 0 \quad \Rightarrow \quad C_p dT = -h\,dp$$

$$dS = 0 \quad \Rightarrow \quad C_p\frac{dT}{T} + \frac{h}{T}dp = 0 \quad \Rightarrow \quad C_p\frac{dT}{T} = -\frac{h}{T}dp$$

Where $C_p = a + bT + cT^2$, $h = -V$, and $\frac{V}{T} = \frac{R}{p}$. Thus:

$$C_p\frac{dT}{T} = -\frac{h}{T}dp \quad \Leftrightarrow \quad (a+bT+cT^2)\frac{dT}{T} = \frac{V}{T}dp \quad \Leftrightarrow \quad (a+bT+cT^2)\frac{dT}{T} = \frac{R}{p}dp$$

$$\Rightarrow a\int_1^2 \frac{dT}{T} + b\int_1^2 dT + c\int_1^2 T\,dT = R\int_1^2 \frac{dp}{p}$$

$$\Leftrightarrow \quad R\ln\left(\frac{p}{p_0}\right) = a\ln\left(\frac{T}{T_0}\right) + b(T-T_0) + \frac{1}{2}c(T^2 - T_0^2)$$

$$\Rightarrow \ln\left(\frac{p}{p_0}\right)^R = \ln\left(\frac{T}{T_0}\right)^a + b(T-T_0) + \frac{1}{2}c(T^2 - T_0^2)$$

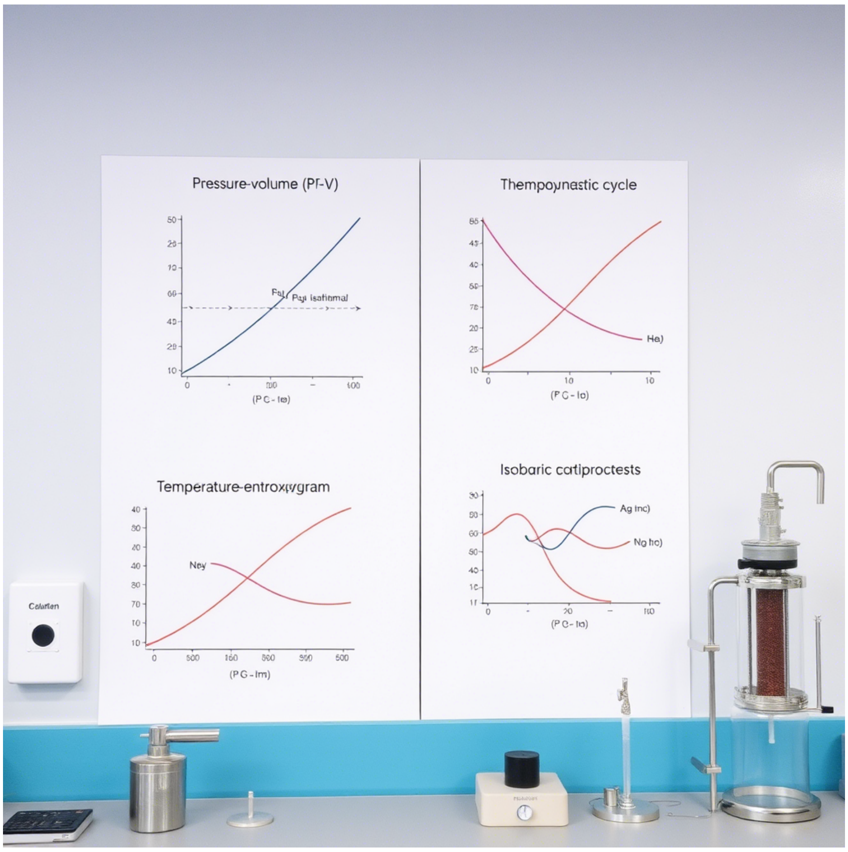
Taking the exponential of the last expression:

$$\left(\frac{p}{p_0}\right)^R = \left(\frac{T}{T_0}\right)^a e^{b(T-T_0)+\frac{1}{2}c(T^2-T_0^2)} \quad \Rightarrow \quad \frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{a}{R}} e^{\frac{1}{R}\left[b(T-T_0)+\frac{1}{2}c(T^2-T_0^2)\right]}$$

$$\Rightarrow p = p_0 \left(\frac{T}{T_0}\right)^{\frac{a}{R}} e^{\frac{1}{R}\left[b(T-T_0)+\frac{1}{2}c(T^2-T_0^2)\right]}$$

Numerical application:

$$p = 1\left(\frac{1000}{273}\right)^{\frac{26.2}{8.32}} e^{\frac{1}{8.32}\left[0.011493(1000-273)-\frac{1}{2}\cdot 3.222\cdot 10^{-6}(1000^2-273^2)\right]} = 136.066\,\mathrm{Pa}$$



Continuous assessments and examination.

0.1 Continuous assessments.

0.1.1 Thermodynamics MCQ 1

Question 1: The First Law of Thermodynamics expresses:

a) Conservation of energy in a thermodynamic system.
b) The irreversibility of natural processes.
c) The equivalence between heat and work.
d) The increase of entropy in an isolated system.

Question 2: An isothermal process is characterized by:

a) Constant pressure.
b) Constant temperature.
c) Constant volume.
d) Zero heat exchange.

Question 3: The efficiency of a Carnot engine depends on:

a) The working substance used.
b) The temperatures of the hot and cold reservoirs.
c) The engine's size.
d) The pressure difference in the cycle.

Question 4: Entropy is a measure of:

a) The internal energy of a system.
b) The disorder or randomness in a system.
c) The heat capacity at constant volume.
d) The enthalpy of a system.

Question 5: In an adiabatic process:

a) The temperature remains constant.
b) No heat is exchanged with the surroundings.
c) The pressure remains constant.
d) The volume remains constant.

0.1.2 Thermodynamics MCQ 2

1. State the First Law of Thermodynamics.
2. The differential of a function f is said to be an exact total differential if it satisfies the following condition:

   ____

3. Write the relation between P, V, and T for an ideal gas.

   ____

4. What are the three modes of heat transfer?

   ____

5. What is the relationship between the heat flux density vector and the thermal flux?

   ____

6. What is the meaning of the negative sign () in Fourier's Law?

   ____

7. What is the expression for the heat diffusion equation?

   ____

8. What is the unit of thermal conductivity?

   ____

9. What is the relation that gives the thermal resistance?

   ____

0.1.3 Thermodynamics MCQ 3

1. What is thermodynamics?
    - (a) The study of material properties
    - (b) The study of heat transfer processes
    - (c) The study of fluid dynamics
    - (d) The study of electrical properties of materials
2. What is a thermodynamic system?
    - (a) A set of mathematical equations
    - (b) A device for measuring temperature
    - (c) An object in motion
    - (d) A collection of matter and energy with which thermal exchanges are studied
3. What is the first law of thermodynamics, also known as the law of energy conservation?
    - (a) Energy cannot be created or destroyed, only transformed
    - (b) Heat moves from a hot body to a cold body
    - (c) The pressure and volume of a gas are inversely proportional
    - (d) The temperature of a gas is directly proportional to its pressure
4. What is an isothermal transformation?
    - (a) A transformation where the temperature remains constant
    - (b) A transformation where the pressure remains constant
    - (c) A transformation where the volume remains constant
    - (d) A transformation where the internal energy of the system remains constant
5. What is entropy?
    - (a) A measure of the internal energy of a system
    - (b) A measure of the temperature of a system
    - (c) A measure of disorder or chaos in a system
6. What is the formula for calculating the work done by a gas during a reversible isothermal expansion?
    - (a) $W = P\Delta V$
    - (b) $W = \Delta T$
    - (c) $W = \Delta U$
    - (d) $W = Q - \Delta U$
7. What is latent heat?
    - (a) The heat required to increase the temperature of a substance

   (b) The heat required to change the state of a substance without changing its temperature
   (c) The heat required to melt a substance
   (d) The heat required to vaporize a substance

8. What is a thermodynamic cycle?
   (a) A process where the temperature remains constant
   (b) A process where the volume remains constant
   (c) A process where the pressure remains constant
   (d) A process where a system returns to its initial state after undergoing a series of transformations

9. How is the coefficient of thermal expansion defined?
   (a) The coefficient of thermal expansion is defined as the change in volume of a body for a given change in temperature.
   (b) The coefficient of thermal expansion is defined as the change in mass of a body for a given change in temperature.
   (c) The coefficient of thermal expansion is defined as the change in length of a body for a given change in temperature.

10. What is an adiabatic process?
   (a) A process where the temperature remains constant
   (b) A process where the pressure remains constant
   (c) A process where no heat exchange occurs with the surroundings
   (d) A process where the volume remains constant

11. What is the formula for calculating the change in internal energy ΔU of a system?
   (a) $\Delta U = Q - W$
   (b) $\Delta U = Q + W$
   (c) $\Delta U = Q/W$
   (d) $\Delta U = Q \times W$

12. What is a closed system in thermodynamics?
   (a) A system that exchanges both matter and energy with the surroundings
   (b) A system that exchanges only energy with the surroundings
   (c) A system that exchanges neither matter nor energy with the surroundings
   (d) A system that exchanges only matter with the surroundings

13. What is an open system in thermodynamics?
   (a) A system that exchanges both matter and energy with the surroundings

   (b) A system that exchanges only energy with the surroundings
   (c) A system that exchanges neither matter nor energy with the surroundings
   (d) A system that exchanges only matter with the surroundings

14. What is a heat engine?
    (a) A device that converts electrical energy into thermal energy
    (b) A device that converts thermal energy into mechanical energy
    (c) A device that converts thermal energy into electrical energy
    (d) A device that converts mechanical energy into thermal energy

15. What is the efficiency of a heat engine?
    (a) The amount of heat produced by the engine
    (b) The amount of electrical energy produced by the engine
    (c) The ratio of useful energy produced to the total energy supplied to the engine
    (d) The amount of fuel consumed by the engine

16. What is the Carnot cycle?
    (a) A reversible thermodynamic cycle consisting of two isothermal and two adiabatic transformations
    (b) An irreversible thermodynamic cycle consisting of two isothermal and two adiabatic transformations
    (c) A reversible thermodynamic cycle consisting of two isobaric and two isochoric transformations
    (d) An irreversible thermodynamic cycle consisting of two isobaric and two isochoric transformations

17. What is the critical temperature of a substance?
    (a) The temperature at which a substance changes state
    (b) The temperature at which a substance reaches its melting point
    (c) The temperature at which a substance reaches its boiling point
    (d) The temperature at which a substance reaches its triple point

18. What is heat transfer by conduction?
    (a) Heat transfer by the movement of a fluid
    (b) Heat transfer by direct contact between two objects at different temperatures
    (c) Heat transfer by electromagnetic radiation

19. What is heat transfer by convection?
    (a) Heat transfer by the movement of a fluid

   (b) Heat transfer by direct contact between two objects at different temperatures
   (c) Heat transfer by electromagnetic radiation

20. What is the heat transfer mechanism involved in heating a room with an electric radiator?
    (a) Conduction
    (b) Convection
    (c) Radiation

21. What is the heat transfer mechanism involved in cooling a hot object in ambient air?
    (a) Conduction
    (b) Convection
    (c) Radiation

22. What is thermal conductivity?
    (a) The ability of a material to transfer heat by conduction
    (b) The ability of a material to transfer heat by convection
    (c) The ability of a material to emit electromagnetic radiation

23. The number of particles in a gas is measured in:
    (a) Grams
    (b) Moles
    (c) Liters
    (d) Joules

24. Temperature is a measure of:
    (a) The momentum of a gas
    (b) The average kinetic energy of the particles in a gas
    (c) The speed of the particles in a gas
    (d) The pressure of a gas

25. The pressure of a gas is directly proportional to:
    (a) The volume of the gas
    (b) The number of particles in the gas
    (c) The speed of the particles in the gas
    (d) The temperature of the gas

0.1.4 Thermodynamics MCQ 4

1. Temperature is a parameter
   (a) Extensive
   (b) Intensive
2. Pressure is a parameter
   (a) Extensive
   (b) Intensive
3. Volume is a parameter
   (a) Extensive
   (b) Intensive
4. A closed system exchanges with its surroundings
   (a) Only matter
   (b) Only energy
   (c) Both matter and energy
   (d) Neither matter nor energy
5. An isolated system exchanges with its surroundings
   (a) Only matter
   (b) Only energy
   (c) Both matter and energy
   (d) Neither matter nor energy
6. What are the different types of transformations?
   ____
7. A reversible transformation is
   (a) Fast
   (b) Slow
8. An irreversible transformation is
   (a) Fast
   (b) Slow
   (c) Quasi-static
   (d) Natural
9. An isochoric transformation occurs at
   (a) Constant pressure
   (b) Constant volume

   (c) Constant temperature
   (d) Constant heat quantity

10. Write the mathematical expression for dV as a function of T and P.

    ____

11. Write the equation of state for an ideal gas.

    ____

12. In the ideal gas model, molecules are considered as
    (a) Point particles
    (b) Masses with spatial dimensions
13. Internal energy U is a
    (a) State function
    (b) Physical quantity
14. Write the first principle of thermodynamics.

    ____

15. What are the different modes of heat transfer?

    ____

16. Thermal conduction is
    (a) Accompanied by the displacement of matter
    (b) Occurs without the displacement of matter
17. Thermal convection is
    (a) Accompanied by the displacement of matter
    (b) Occurs without the displacement of matter
18. Heat transfer always occurs from
    (a) Hot to cold
    (b) Cold to hot
    (c) All answers are incorrect
19. During the compression of an ideal gas (e.g., in a piston)
    (a) It receives work
    (b) It supplies work
    (c) The work is negative
    (d) The work is positive

0.1.5 Thermodynamics MCQ 5

1. Pressure P is a state function, therefore:
   - (a) ☐ The differential dP is exact
   - (b) ☐ The differential dP is inexact
   - (c) ☐ A differential can be exact or inexact
2. The internal energy of an ideal gas is:
   - (a) ☐ A function of temperature T only
   - (b) ☐ A function of both T and P
   - (c) ☐ Never a function of volume
3. The change in internal energy during adiabatic compression of a gas is expressed by work W and equals:
   - (a) ☐ $\Delta U = W$
   - (b) ☐ $\Delta U = -W$
   - (c) ☐ $\Delta U = 0$
4. Fourier's Law describes the relationship between:
   - (a) ☐ Heat flux and temperature gradient
   - (b) ☐ Flux density and heat flux
   - (c) ☐ Pressure and volume of a gas
5. Fourier's Law applies to heat transfer by:
   - (a) ☐ Convection
   - (b) ☐ Conduction
   - (c) ☐ Both convection and conduction
   - (d) ☐ Fluid motion
6. Thermal conductivity is:
   - (a) ☐ A negative property of the medium
   - (b) ☐ A positive property of the medium
   - (c) ☐ An integer that can degrade
7. The phenomenon of thermal convection:
   - (a) ☐ Involves displacement of matter
   - (b) ☐ Occurs only in solids
   - (c) ☐ Occurs in fluids
8. The direction of the heat flux density vector indicates:
   - (a) ☐ The direction of heat transfer (from hot to cold)

   (b) ☐ Increasing temperature
   (c) ☐ The direction opposite to the temperature gradient

9. The heat flux density vector and temperature gradient vector have the same direction:
   (a) ☐ True
   (b) ☐ False

10. The heat equation describes temperature variation:
    (a) ☐ In space
    (b) ☐ In time
    (c) ☐ In both space and time

0.1.6 Thermodynamics MCQ 6

1. What are the three modes of heat transfer?
   (a) Convection, radiation, diffusion
   (b) Conduction, radiation, advection
   (c) Convection, conduction, radiation
2. Which of these heat transfer modes requires a material medium to propagate?
   (a) Convection
   (b) Radiation
   (c) Conduction
3. Which heat transfer mode occurs in moving fluids?
   (a) Conduction
   (b) Radiation
   (c) Convection
4. In which direction does heat transfer occur during conduction?
   (a) From hot zone to cold zone
   (b) From cold zone to hot zone
   (c) In both directions simultaneously
5. Which heat transfer mode occurs through electromagnetic wave emission?
   (a) Conduction
   (b) Radiation
   (c) Convection
6. Which heat transfer mode is most significant in a vacuum?
   (a) Conduction
   (b) Radiation
   (c) Convection
7. What is the greenhouse effect?
   (a) A thermal conduction phenomenon
   (b) A thermal convection phenomenon
   (c) A thermal radiation phenomenon
8. How does convective heat transfer occur?
   (a) Through movement of hot and cold particles
   (b) Through direct contact between objects
   (c) Through electromagnetic wave emission

9. Which heat transfer mode occurs in solids?
   (a) Conduction
   (b) Radiation
   (c) Convection
10. How does radiative heat transfer occur?
   (a) Through movement of hot and cold particles
   (b) Through direct contact between objects
   (c) Through electromagnetic wave emission
11. Which heat transfer mode can be reduced by adding insulating material?
   (a) Conduction
   (b) Radiation
   (c) Convection
12. What is Fourier's Law?
   (a) A law describing radiative heat transfer
   (b) A law describing convective heat transfer
   (c) A law describing conductive heat transfer
13. Which heat transfer mode is often responsible for heat losses in heating and cooling systems?
   (a) Conduction
   (b) Radiation
   (c) Convection
14. Which heat transfer mode can occur through a solid wall between two media at different temperatures?
   (a) Conduction
   (b) Radiation
   (c) Convection
15. In which medium is convective heat transfer most efficient?
   (a) Solids
   (b) Liquids
   (c) Gases
16. Which material is the best thermal conductor?
   (a) Air
   (b) Water
   (c) Copper

17. How does a material's thermal conductivity affect heat transfer through it?
    (a) Higher conductivity facilitates heat transfer
    (b) Higher conductivity impedes heat transfer
    (c) Conductivity doesn't affect heat transfer
18. How does material thickness affect heat transfer through it?
    (a) Thicker material facilitates heat transfer
    (b) Thicker material impedes heat transfer
    (c) Thickness doesn't affect heat transfer
19. How does adding insulation affect heat transfer through a wall?
    (a) Increases heat transfer
    (b) Decreases heat transfer
    (c) Doesn't affect heat transfer
20. Match the following letters (a, b, c, d, e, g, h) with their correct labels in both diagrams:
    - Thermometer
    - Stirrer
    - Support
    - Thermostat
    - Goniometer
    - Metal rod
    - Thermocouple
    - Caliper
    - Dilatometer (comparator)
    - Immersion heater
    - Electrical resistance
    - Device for measuring linear expansion coefficient of metals
    - Stopwatch
    - Beaker
    - Current generator
    - Electronic thermometer
    - Voltage generator
    - Device for measuring thermal conductivity of metals
    - Calorimeter
    - Balance
    - Graduated ruler

0.1.7 Thermodynamics MCQ 7

1. Heat is a:
   (a) ☐ State function
   (b) ☐ Physical quantity
   (c) ☐ Other: ______________
2. Internal energy is a:
   (a) ☐ State function
   (b) ☐ Physical quantity
   (c) ☐ Differential form
   (d) ☐ Exact differential
   (e) ☐ Other: ______________
3. The internal energy of an ideal gas depends on:
   (a) ☐ Volume
   (b) ☐ Pressure
   (c) ☐ Temperature only
   (d) ☐ Volume and pressure
   (e) ☐ Temperature and volume
   (f) ☐ Other: ______________
4. $PV = nRT$ is the equation of a:
   (a) ☐ Ideal gas
   (b) ☐ Real gas
   (c) ☐ Other: ______________
5. If pressure P is a state function, then its differential is:
   (a) ☐ Inexact
   (b) ☐ Exact
   (c) ☐ Other: ______________
6. Thermal conductivity is a:
   (a) ☐ Positive constant
   (b) ☐ Negative constant
   (c) ☐ Non-constant
   (d) ☐ Other: ______________
7. Fourier's Law applies to:
   (a) ☐ Liquids

   (b) ☐ Solids
   (c) ☐ Gases
   (d) ☐ All of the above

8. When a radiator heats a house, the observed heat transfer modes are:
   (a) ☐ Thermal conduction
   (b) ☐ Thermal convection
   (c) ☐ Thermal radiation
   (d) ☐ All three modes simultaneously

9. Heat flux density is:
   (a) ☐ Heat flow per unit area
   (b) ☐ Heat quantity per unit time
   (c) ☐ Heat received by the system
   (d) ☐ Other: ____________

10. By analogy with electricity, thermal resistance is equivalent to electrical resistance, which is the ratio between:
    (a) ☐ Temperature difference and heat flux
    (b) ☐ Temperature difference and heat flux density
    (c) ☐ Other: ____________

0.1.8 Thermodynamics exercice test 1

A gas with the equation of state $V = V(T, P)$ has the isobaric thermal expansion coefficient $\alpha = \frac{R}{PV}$ and the isothermal compressibility coefficient $\chi_T = \frac{RT}{VP^2}$, where R is the ideal gas constant (Mayer's constant).

1. Express the differential dV of the gas volume in terms of α and χ_T as a function of dT and dP.
2. By integration, deduce the equation of state of the gas, given that when $V = 2b$, the condition $T = bP = R$ holds.

Recall the definitions of the coefficients α *and* χ_T*:*

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P, \quad \chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

Solution

1. The expression for the differential dV of the gas volume in terms of α and χ_T:

$$dV = V\alpha\, dT - V\chi_T\, dP$$

2. The equation of state of the gas:

$$V(T, P) = \frac{RT}{P} + C$$

Applying the given condition $V = 2b$ when $T = bP = R$, we find the integration constant C, leading to:

$$V(T, P) = \frac{RT}{P} + b$$

Solution: Thermodynamics exercice test 1

1. **Expression for the differential dV**
   The total differential of $V(T,P)$ is given by:
   $$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP$$
   Using the definitions of α and χ_T, we can rewrite the partial derivatives:
   $$\left(\frac{\partial V}{\partial T}\right)_P = V\alpha \quad \text{and} \quad \left(\frac{\partial V}{\partial P}\right)_T = -V\chi_T$$
   Substituting these into the expression for dV, we obtain:
   $$\boxed{dV = V\alpha\, dT - V\chi_T\, dP}$$

2. **Integration to find the equation of state**
   Given the expressions for α and χ_T:
   $$\alpha = \frac{R}{PV} \quad \text{and} \quad \chi_T = \frac{RT}{VP^2}$$
   Substitute these into the differential dV:
   $$dV = V\left(\frac{R}{PV}\right) dT - V\left(\frac{RT}{VP^2}\right) dP = \frac{R}{P}dT - \frac{RT}{P^2}dP$$
   To integrate, rewrite dV as:
   $$dV = R\left(\frac{dT}{P} - \frac{T}{P^2}dP\right)$$
   Notice that the term in parentheses is the exact differential of $\frac{T}{P}$:
   $$d\left(\frac{T}{P}\right) = \frac{dT}{P} - \frac{T}{P^2}dP$$
   Thus:
   $$dV = R\, d\left(\frac{T}{P}\right)$$
   Integrate both sides:
   $$V = R\left(\frac{T}{P}\right) + C$$
   where C is the integration constant.

   Determine the constant C using the given condition: When $V = 2b$, $T = bP = R$. From $T = bP$, we have $P = \frac{T}{b}$. Substituting $T = R$:
   $$P = \frac{R}{b}$$

Now, substitute $V = 2b$, $T = R$, and $P = \frac{R}{b}$ into the equation:

$$2b = R\left(\frac{R}{\frac{R}{b}}\right) + C = R\left(\frac{R \cdot b}{R}\right) + C = Rb + C$$

Solving for C:

$$C = 2b - Rb = b(2 - R)$$

However, since $T = bP = R$, we have $b = \frac{R}{P}$. But from the earlier substitution, $P = \frac{R}{b}$, which is consistent. Thus, the simplest form is obtained by recognizing that the condition $T = bP = R$ implies b is a constant with appropriate units, and the final equation of state is:

$$\boxed{V(T, P) = \frac{RT}{P} + b}$$

0.1.9 Thermodynamics exercice test 2

Consider the differentials:

$$dH = C_P\,dT + (h+V)\,dP \quad \text{and} \quad dS = \frac{C_P}{T}\,dT + \frac{h}{T}\,dV$$

where C_P and h are the calorimetric coefficients related to the single-phase system under study.

1. By explicating the relations imposed by the fact that dH and dS are exact total differentials, determine the coefficient h. Deduce the coefficient $\left(\frac{\partial C_P}{\partial P}\right)_T$.
2. We now consider the equation of state $PV - nRT = 0$, where n and R are constants. Compute h; show that C_P does not depend on P. Calculate the state functions H and S, assuming that C_P does not depend on T.
3. We now consider the equation of state $P(V - nb) - nRT = 0$. Revisit question 2 under the same assumptions.

Solution: Thermodynamics exercice test 2

Part 1: General Relations from Exact Differentials

Given the differentials:

$$dH = C_P\, dT + (h + V)\, dP \quad (1)$$

$$dS = \frac{C_P}{T}\, dT + \frac{h}{T}\, dP \quad (2)$$

a. Determining coefficient h

For dH to be an exact differential, we must have:

$$\left(\frac{\partial C_P}{\partial P}\right)_T = \left(\frac{\partial (h+V)}{\partial T}\right)_P \quad (3)$$

For dS to be an exact differential, we must have:

$$\left(\frac{\partial (C_P/T)}{\partial P}\right)_T = \left(\frac{\partial (h/T)}{\partial T}\right)_P \quad (4)$$

Developing equation (4):

$$\frac{1}{T}\left(\frac{\partial C_P}{\partial P}\right)_T = \frac{1}{T}\left(\frac{\partial h}{\partial T}\right)_P - \frac{h}{T^2}$$

$$\left(\frac{\partial C_P}{\partial P}\right)_T = \left(\frac{\partial h}{\partial T}\right)_P - \frac{h}{T} \quad (5)$$

From equation (3) and the thermodynamic identity:

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T = -\frac{h}{T}$$

Thus:

$$h = -T\left(\frac{\partial V}{\partial T}\right)_P \quad (6)$$

b. Deduction of $(\frac{\partial C_P}{\partial P})_T$

Substituting (6) into (5):

$$\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P + \left(\frac{\partial V}{\partial T}\right)_P - \left(\frac{\partial V}{\partial T}\right)_P$$

$$\boxed{\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P}$$

Part 2: Ideal Gas Case $PV = nRT$

a. Calculation of h

From the equation of state:

$$V = \frac{nRT}{P} \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \frac{nR}{P}$$

Using equation (6):

$$h = -T\left(\frac{nR}{P}\right) = -\frac{nRT}{P} = -V$$

Thus:

$$\boxed{h = -V}$$

b. Pressure Independence of C_P

From 1.2:

$$\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P = 0$$

since V is linear in T. Therefore:

$$\boxed{C_P \text{ does not depend on } P}$$

c. State Functions H and S

Assuming C_P constant:

For enthalpy:

$$dH = C_P dT + (h + V)dP = C_P dT + 0 \cdot dP$$

$$H(T, P) = C_P T + H_0$$

For entropy:

$$dS = \frac{C_P}{T}dT + \frac{h}{T}dP = \frac{C_P}{T}dT - \frac{nR}{P}dP$$
$$S(T, P) = C_P \ln T - nR \ln P + S_0$$

Thus:

$$\boxed{H(T) = C_P T + H_0}$$

$$\boxed{S(T, P) = C_P \ln T - nR \ln P + S_0}$$

Part 3: Modified Gas Equation $P(V - nb) = nRT$

a. Calculation of h

From the equation of state:

$$V = \frac{nRT}{P} + nb \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \frac{nR}{P}$$

Thus h remains:

$$h = -T\left(\frac{nR}{P}\right) = -\frac{nRT}{P} = -(V - nb)$$

$$\boxed{h = -(V - nb)}$$

b. Pressure Independence of C_P

Again:

$$\left(\frac{\partial^2 V}{\partial T^2}\right)_P = 0 \Rightarrow \left(\frac{\partial C_P}{\partial P}\right)_T = 0$$

$$\boxed{C_P \text{ remains independent of } P}$$

c. State Functions H and S

For enthalpy:

$$dH = C_P dT + (h + V)dP = C_P dT + nbdP$$

$$H(T, P) = C_P T + nbP + H_0$$

For entropy:

$$dS = \frac{C_P}{T}dT + \frac{h}{T}dP = \frac{C_P}{T}dT - \frac{nR}{P}dP$$

(same as ideal gas case)

Thus:

$$\boxed{H(T, P) = C_P T + nbP + H_0}$$

$$\boxed{S(T, P) = C_P \ln T - nR \ln P + S_0 \text{ (same as ideal gas)}}$$

The entropy expression remains identical to the ideal gas case because the equation of state modification only affects the volume available to molecules (through b) but not the temperature dependence of pressure.

0.1.10 Thermodynamics exercise test 3

1. Ideal Gas – Mayer's Relation

Show that for an ideal gas, the difference between the heat capacities at constant pressure (C_P) and at constant volume (C_V) satisfies the relation:

$$C_P - C_V = nR$$

where n is the number of moles and R is the ideal gas constant.

2. Thermoelastic Coefficients of an Ideal Gas

For an ideal gas obeying the equation of state $PV = nRT$, express:

- The isobaric expansion coefficient $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$
- The isothermal compressibility coefficient $\chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$

3. Real Gas – Van der Waals Equation

A real gas follows the Van der Waals equation:

$$\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$$

where V_m is the molar volume, a and b are constants.
Calculate the isobaric expansion coefficient α for this gas.

4. General Relation Between C_P and C_V

Using general thermodynamic relations, show that:

$$C_P - C_V = \frac{TV\alpha^2}{\chi_T}$$

where α is the isobaric expansion coefficient and χ_T is the isothermal compressibility coefficient.

5. Variation of C_V for a Real Gas

Show that, for a real gas, the heat capacity at constant volume C_V depends only on temperature if the equation of state has the form:

$$P = Tf(V) + g(V)$$

where $f(V)$ and $g(V)$ are arbitrary functions of volume.

Solution: Thermodynamics exercice test 3

1. Mayer's Relation for an Ideal Gas

For an ideal gas, we start from the definitions of heat capacities:

$$C_P = \left(\frac{\partial H}{\partial T}\right)_P \quad \text{and} \quad C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

With enthalpy $H = U + PV$ and the equation of state $PV = nRT$, we have:

$$H = U + nRT$$

Differentiating:

$$dH = dU + nR\,dT$$

Thus:

$$C_P\,dT = C_V\,dT + nR\,dT$$

Finally:

$$\boxed{C_P - C_V = nR}$$

2. Thermoelastic Coefficients of an Ideal Gas

Isobaric Expansion Coefficient α :

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

From $PV = nRT$, we get $V = \frac{nRT}{P}$ so:

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{nR}{P}$$

Thus:

$$\boxed{\alpha = \frac{1}{V}\cdot\frac{nR}{P} = \frac{1}{T}}$$

Isothermal Compressibility Coefficient χ_T :

$$\chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

From $V = \frac{nRT}{P}$, we have:

$$\left(\frac{\partial V}{\partial P}\right)_T = -\frac{nRT}{P^2}$$

Therefore:

$$\boxed{\chi_T = \frac{1}{P}}$$

3. Coefficient α for a Van der Waals Gas

The equation of state:

$$\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$$

We differentiate at constant pressure:

$$\left(0 - \frac{2a}{V_m^3} dV_m\right)(V_m - b) + \left(P + \frac{a}{V_m^2}\right) dV_m = R\,dT$$

Rearranging:

$$\left[P + \frac{a}{V_m^2} - \frac{2ab}{V_m^3} + \frac{2a}{V_m^2}\right] dV_m = R\,dT$$

The coefficient α becomes:

$$\boxed{\alpha = \frac{R}{V_m\left[P + \frac{3a}{V_m^2} - \frac{2ab}{V_m^3}\right]}}$$

4. General Relation Between C_P and C_V

Starting from the differential of entropy $S(T, V)$:

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

Using Maxwell's relations and the definitions of heat capacities, we obtain:

$$C_P - C_V = T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$$

Introducing α and χ_T, we arrive at:

$$\boxed{C_P - C_V = \frac{TV\alpha^2}{\chi_T}}$$

5. Independence of C_V with Respect to Volume

For a real gas with $P = Tf(V) + g(V)$, we calculate:

$$\left(\frac{\partial P}{\partial T}\right)_V = f(V)$$

From the relation:

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V = 0$$

because P is linear in T.

Thus:

$$\boxed{C_V \text{ depends only on temperature}}$$

0.2 Examination - Thermodynamics.

0.2.1 Final Examination - Thermodynamics 1

Exercise 1 (5 points)

Let v be the specific volume ($\mathrm{m^3\,kg^{-1}}$) of an ideal gas with molar mass M.

1. Show that the equation of state can be written as $Pv = rT$. Specify the expression for r and its units.
2. Calculate the value of r for dioxygen.
3. Deduce the specific volume of dioxygen at $300\,\mathrm{K}$ and $1\,\mathrm{bar}$.

Given: $M_O = 16\,\mathrm{g\,mol^{-1}}$; $R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$;
$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $1\,\mathrm{bar} = 10^5\,\mathrm{Pa}$.

Exercise 2 (7 points)

A futuristic dwelling consists of a hemispherical wall with inner radius R_1 and outer radius R_2, placed on horizontal ground. Let O be the center of the complete sphere and M a point in the wall ($R_1 \leq r = OM \leq R_2$). The wall material is homogeneous and isotropic with mass density μ and specific heat capacity C.

A constant temperature T_1 is maintained inside the dwelling and on the inner wall surface, while temperature $T_2 < T_1$ is maintained in the outside air (thermal reservoir) and on the outer wall surface.

1. Write the partial differential equation satisfied by the temperature $T(r,t)$ at any point in the material, assuming rotational symmetry about any axis through O. Recall the Laplacian in spherical coordinates:
$$\Delta U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial U}{\partial r}\right)$$
2. Determine the steady-state temperature distribution within the material.
3. Express the heat flux escaping from the dwelling, neglecting ground effects.
4. Calculate the thermal resistance of the dwelling, still neglecting ground effects.

Exercise 3 (8 points)

Given:
$$\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}, \quad \int_0^\infty x^4 e^{-ax^2} dx = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}}$$

1. In an oven, cesium-133 behaves as a monoatomic ideal gas. A volume V contains N atoms of mass m (number density $n_0 = N/V$) at temperature T.
   (a) Explain the microscopic meaning of ”ideal gas”.
   (b) Express the kinetic energy E of a monoatomic ideal gas atom as a function of its velocity.

2. Assuming Maxwellian statistics, the number of atoms in volume dV with speed between v and $v + dv$ is:

$$dN = A \exp\left(-\frac{E}{kT}\right) v^2 dv dV$$

   (a) Express the total number of atoms N in volume V as a function of A.

   (b) Calculate the root-mean-square speed v_q.

3. Define the internal energy U of this ideal gas in volume V and express it as a function of absolute temperature T.

4. Using the ideal gas law, show that the pressure p can be written as:

$$p = \frac{1}{3} n_0 m v_q^2$$

0.2.2 Final Examination - Thermodynamics 2

Exercise 1 (6 points)

The partial derivatives of the internal energy $U(S, V, N)$ for a system with variable number of moles N are given by:

$$\left(\frac{\partial U}{\partial S}\right)_{V,N} = T; \quad \left(\frac{\partial U}{\partial V}\right)_{S,N} = -P; \quad \left(\frac{\partial U}{\partial N}\right)_{S,V} = \mu.$$

1. Write the differential form of U.
2. Establish the expression $U = TS - PV + \mu N$.
3. Deduce the Gibbs-Duhem relation: $SdT - VdP + Nd\mu = 0$.

Exercise 2 (7 points)

A solid cylindrical copper rod of axis (OX), length l, radius a, and thermal conductivity K is in contact at one end $(x = 0)$ with a heat exchanger at temperature T_0. Its lateral surface and other end $(x = l)$ are in contact with a fluid at constant temperature T_e $(T_0 > T_e)$.

- We assume steady-state conditions and that the radial temperature gradient within the rod is sufficiently weak to consider the temperature $T(x)$ uniform in the cross-section at position x. The rod exhibits thermal losses at its surface in contact with the fluid, per unit time and surface area, given by $h(T(x) - T_e)$, where $T(x)$ is the temperature at the surface point considered and h is a constant coefficient.

1. Determine the temperature distribution $T(x)$ within the rod.
2. Calculate $T(l)$.

Given: $K = 389\,\mathrm{W\,m^{-1}\,K^{-1}}$, $h = 155\,\mathrm{W\,m^{-2}\,K^{-1}}$, $a = 1\,\mathrm{mm}$, $T_0 = 340\,\mathrm{K}$, $T_e = 300\,\mathrm{K}$, $l = 10\,\mathrm{cm}$.

Exercise 3 (7 points)

We consider only steady-state regimes, independent of time.

The interior of a room is separated from the exterior by a glass wall of surface area S, orthogonal to the (OX) axis, with glass thermal conductivity K. Its inner and outer faces are at temperatures T_i and T_e respectively $(T_e < T_i)$.

1. The wall consists of a single glass pane of thickness e.
    - Evaluate the heat flux Φ_1 exiting the room through this wall as a function of K, S, e, T_i, and T_e.
    - Calculate the thermal resistance R_{th} of the glass wall.
2. The wall consists of two glass panes of equal thickness e, separated by a layer of air of thickness e' with thermal conductivity K'. (Only conduction is considered.)

- Evaluate the heat flux Φ_2 exiting the room, then compute the ratio Φ_2/Φ_1.

3. **Numerical Application:** $T_e = 270\,\mathrm{K}$, $T_i = 292\,\mathrm{K}$, $e' = e = 3\,\mathrm{mm}$, $K = 1.2\,\mathrm{W\,m^{-1}\,K^{-1}}$, $K' = 0.025\,\mathrm{W\,m^{-1}\,K^{-1}}$.

   Calculate Φ_2/Φ_1 and the temperatures T_1 and T_2 at the facing surfaces of the two panes.

4. Graphically represent the temperature variations as a function of x in the double-glazing system.

0.2.3 Final Examination - Thermodynamics 3

Exercise 1 (6 points)

The partial derivatives of the internal energy $U(S,V,N)$ for a system with variable number of moles N are given by:

$$\left(\frac{\partial U}{\partial S}\right)_{V,N} = T; \quad \left(\frac{\partial U}{\partial V}\right)_{S,N} = -P; \quad \left(\frac{\partial U}{\partial N}\right)_{S,V} = \mu$$

1. Write the differential form of U.
2. Establish the expression: $U = TS - PV + \mu N$.
3. Deduce the Gibbs-Duhem relation: $SdT - VdP + Nd\mu = 0$

Exercise 2 (7 points)

A solid cylindrical copper rod of axis (OX), length l, radius a, and thermal conductivity K has one end $(x = 0)$ in contact with a heat exchanger at temperature T_0, while its lateral surface and other end $(x = l)$ are in contact with a fluid at constant temperature T_e (with $T_0 > T_e$).

Under steady-state conditions, assuming the radial temperature gradient within the rod is sufficiently weak to consider the temperature $T(x)$ uniform in the cross-section at abscissa x. The rod exhibits thermal losses at its surface in contact with the fluid, per unit time and surface area, equal to $h(T(x) - T_e)$, where h is a constant coefficient.

- Determine the temperature distribution $T(x)$ within the rod.
- Calculate $T(l)$.

Given: $K = 389\,\mathrm{W\cdot m^{-1}\cdot K^{-1}}$, $h = 155\,\mathrm{W\cdot m^{-2}\cdot K^{-1}}$, $a = 1\,\mathrm{mm}$, $T_0 = 340\,\mathrm{K}$, $T_e = 300\,\mathrm{K}$, $l = 10\,\mathrm{cm}$.

Exercise 3 (7 points)

Consider only steady-state conditions, independent of time.

The interior of a room is separated from the exterior by a glass wall of surface area S, orthogonal to the (OX) axis, with glass thermal conductivity K. Its inner and outer faces are at temperatures T_i and T_e respectively $(T_e < T_i)$.

3.1 Single-pane window of thickness e

(a) Evaluate the outgoing thermal flux Φ_1 through this wall as a function of K, S, e, T_i, and T_e.

(b) Calculate the thermal resistance R_{th} of the glass wall.

3.2 Double-pane window

The wall consists of two glass panes of equal thickness e, separated by an air gap of thickness e' with thermal conductivity K'. Consider only conduction.

(a) Evaluate the outgoing thermal flux Φ_2, then compute $\frac{\Phi_2}{\Phi_1}$.

(b) **Numerical application:** $T_e = 270\,\mathrm{K}$; $T_i = 292\,\mathrm{K}$; $e' = e = 3\,\mathrm{mm}$; $K = 1.2\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$; $K' = 0.025\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$.

Compute $\frac{\Phi_2}{\Phi_1}$ and the temperatures T_1 and T_2 at the facing surfaces of the two panes.

Plot the temperature variation as a function of x for the double-glazing system.

0.2.4 Final Examination - Thermodynamics 4

Exercise 1 (7 points)

The partial derivatives of the internal energy $U(S, V, N)$ for a system with variable number of moles N are given by:

$$\left(\frac{\partial U}{\partial S}\right)_{V,N} = T; \quad \left(\frac{\partial U}{\partial V}\right)_{S,N} = -P; \quad \left(\frac{\partial U}{\partial N}\right)_{S,V} = \mu$$

1. Write the differential form of U.
2. Establish the expression: $U = TS - PV + \mu N$.
3. Deduce the Gibbs-Duhem relation: $SdT - VdP + Nd\mu = 0$

Exercise 2 (5 points)

- Verify that the following expression is an exact differential of some function $f(x, y)$:

$$\frac{x}{(x^2 + y^2)^2}dx + \frac{y}{(x^2 + y^2)^2}dy$$

- Determine the function $f(x, y)$.

Exercise 3 (8 points)

Consider only steady-state conditions, independent of time.

The interior of a room is separated from the exterior by a glass wall of surface area S, orthogonal to the (OX) axis, with glass thermal conductivity K. Its inner and outer faces are at temperatures T_i and T_e respectively $(T_e < T_i)$.

3.1 Single-pane window of thickness e (Figure 1)

(a) Evaluate the outgoing thermal flux Φ_1 through this wall as a function of K, S, e, T_i, and T_e.

(b) Calculate the thermal resistance R_{th} of the glass wall.

3.2 Double-pane window (Figure 2)

The wall consists of two glass panes of equal thickness e, separated by an air gap of thickness e' with thermal conductivity K'. Consider only conduction.

(a) Evaluate the outgoing thermal flux Φ_2, then compute $\frac{\Phi_2}{\Phi_1}$.

(b) **Numerical application:**

$$\begin{aligned} T_e &= 270\,\mathrm{K}; \qquad\qquad\qquad T_i = 292\,\mathrm{K}; \\ e' = e &= 3\,\mathrm{mm}; \\ K &= 1.2\,\mathrm{W\,m^{-1}\,K^{-1}}; \\ K' &= 0.025\,\mathrm{W\,m^{-1}\,K^{-1}} \end{aligned}$$

0.2.5 Final Examination - Thermodynamics 5

Exercise 1 (10 points)

Consider the differential forms:

$$dH = C_P dT + (h + V)dP \quad \text{and} \quad dS = \frac{C_P}{T}dT + \frac{h}{T}dP$$

where C_P and h are calorimetric coefficients related to the single-phase system under study.

1. By explicating the relations imposed by the fact that dH and dS are exact differentials, determine the coefficient h. Deduce the coefficient $\left(\frac{\partial C_P}{\partial P}\right)_T$.
2. Consider the equation of state $PV - nRT = 0$ where n and R are constants:
   - Calculate h
   - Show that C_P does not depend on P
   - Compute the state functions H and S assuming C_P does not depend on T
3. Now consider the equation of state $P(V - nb) - nRT = 0$:
   - Repeat question 2 under the same assumptions

Exercise 2 (10 points)

A futuristic dwelling consists of a hemispherical wall with inner radius R_1 and outer radius R_2, placed on horizontal ground. Let O be the center of the complete sphere and M a point in the wall (such that $OM = r$ with $R_1 \leq r \leq R_2$). The wall material is homogeneous and isotropic with mass density μ and specific heat capacity C.

A constant temperature T_1 is maintained inside the dwelling and on the inner wall surface, while a temperature $T_2 < T_1$ is maintained in the outside air (considered as a thermal reservoir) and on the outer wall surface.

1. Write the partial differential equation satisfied by the temperature $T(r, t)$ at any point in the material. Assume rotational symmetry about any axis through O. Recall the expression for the Laplacian of a scalar function $U(r, t)$ in spherical coordinates:

$$\Delta U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial U}{\partial r}\right)$$

2. In steady-state conditions, determine the temperature distribution within the wall.
3. Express the heat flux escaping from the dwelling, neglecting ground effects.
4. What is the thermal resistance of the dwelling, still neglecting ground effects?

0.2.6 Final Examination - Thermodynamics 6

Exercise 1 (10 points)

Consider one mole of gas occupying volume V_m under pressure P at temperature T.

1. Assume these quantities are related by the equation:

   $$\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$$

   where a, b and R are constants.

   Using the intensive or extensive properties of the variables, derive the corresponding equation for n moles.

2. Repeat the question for the equation:

   $$P(V_m - b)\exp\left(\frac{a}{RTV_m}\right) = RT$$

Exercise 2 (10 points)

Given:

$$\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}, \quad \int_0^\infty x^4 e^{-ax^2} dx = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}}$$

1. In an oven, cesium-133 behaves as a monoatomic ideal gas. A volume V contains N atoms of mass m (with number density $n_0 = N/V$) at temperature T.
   - Express the kinetic energy E_c of a monoatomic ideal gas atom as a function of its velocity.
2. Assuming the gas follows Maxwellian statistics: the number of atoms in an infinitesimal volume dV with speed between v and $v + dv$ is:

   $$dN = A\exp\left(-\frac{E_c}{kT}\right) v^2 dv dV$$

   (a) Express the total number of atoms N in volume V as a function of A.

   (b) Calculate the root-mean-square speed v_q using the previous result and the relation $f(v)dv = dN/N$.
3. Define the internal energy U of the ideal gas in volume V and express it as a function of absolute temperature T.
4. Using the ideal gas law, show that the pressure p can be written as:

   $$p = \frac{1}{3} n_0 m v_q^2$$

Note: Boltzmann's constant is $k = R/N_A$ and the number of moles is $n = N/N_A$, where N_A is Avogadro's number.

0.2.7 Final Examination - Thermodynamics 7

Exercise 1 (10 points)

Recall the differential forms of internal energy U, enthalpy H, and entropy S for a gas:

$$dU = C_V dT + (l - P)dV$$
$$dH = C_P dT + (h + V)dP$$
$$dS = \frac{C_V}{T}dT + \frac{l}{T}dV = \frac{C_P}{T}dT + \frac{h}{T}dP$$

where P, V, and T represent pressure, volume, and temperature respectively.

1. What are the coefficients C_V and C_P called?
2. Prove the relations:
$$l = T\left(\frac{\partial P}{\partial T}\right)_V, \quad h = -T\left(\frac{\partial V}{\partial T}\right)_P$$
3. Calculate coefficients l and h for an ideal gas. Show that in this case C_V and C_P depend only on temperature.
4. Prove that for one mole of ideal gas, the coefficients satisfy Mayer's relation:
$$C_P - C_V = R$$
where R is the ideal gas constant.
5. For an ideal gas G with constant C_V and $\gamma = C_P/C_V$, prove that during any isentropic process the pressure and volume are related by Laplace's relation:
$$PV^\gamma = \text{constant}$$
6. For one mole of this ideal gas G, derive general expressions for:
   - Internal energy U
   - Entropy S (first as function of T and V, then as function of T and P)

Exercise 2 (10 points)

One mole of ideal gas with constant specific heat C_V is enclosed in an adiabatic cylinder with a piston. Initial state: (P_0, V_0, T_0). A pressure P_1 is suddenly applied to the piston. Final equilibrium state: (P_1, T_1, V_1).

1. Explain why this transformation is irreversible.
2. Express the work done on the gas in terms of P_1, V_0, and V_1.
3. Express the internal energy change using T_0 and T_1.
4. Determine the final temperature T_1 as function of T_0, P_0, P_1, C_V, and R.
5. What is the final volume V_1? Express in terms of V_0, P_0, P_1, C_V, and R.
6. What limit does V_1 approach as $P_1 \to \infty$?

0.2.8 Final Examination - Thermodynamics 8

Exercise 1 (10 points)

Consider a quantity n of a van der Waals gas with equation of state:

$$\left(p+\frac{n^2a}{V^2}\right)(V-nb)=nRT$$

where a, b and R are constants. Differentiating both sides leads to an equation of the form:

$$Adp+BdV+CdT=0$$

1. Find expressions for A, B, and C.
2. Express the following partial derivatives in terms of A, B, and C:
   - $\left(\frac{\partial V}{\partial T}\right)_p$
   - $\left(\frac{\partial p}{\partial T}\right)_V$
   - $\left(\frac{\partial V}{\partial p}\right)_T$
3. Derive expressions for the thermoelastic coefficients:
   - $\alpha=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$
   - $\beta=\frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V$
   - $\chi_T=-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$

   in terms of p, V, and possibly n, a, b, and R.
4. Derive expressions for these coefficients in the case of an ideal gas.
5. Verify whether the relation $\frac{\alpha}{\beta\chi_T}=p$ holds for the expressions obtained in the previous questions.

Exercise 2 (10 points)

The internal energy U and entropy S of a single-phase system are state functions with differential forms:

$$dU=C_VdT+(l-P)dV$$

$$dS=\frac{C_V}{T}dT+\frac{l}{T}dV$$

where C_V is the isochoric heat capacity and l is the latent heat of expansion. These calorimetric coefficients depend on P, V, T, and n.

1. Derive the relations imposed by the exactness of dU and dS.
2. Express l and $\left(\frac{\partial C_V}{\partial V}\right)_T$ in terms of thermodynamic quantities and their partial derivatives.

3. Consider a closed system of n moles of ideal gas:
    - Determine the expression for l
    - Show that the isochoric heat capacity C_V is independent of V
    - Find expressions for the state functions U and S, assuming C_V is independent of T

0.2.9 Final Examination - Thermodynamics 9

Exercise 1 (5 points)

A gas obeys the van der Waals equation for one mole:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where a and b are positive constants.

1. What are the SI units of a and b?
2. Write the van der Waals equation for n moles.

Exercise 2 (6 points)

A gas with equation of state $V = V(T, P)$ has isobaric thermal expansion coefficient:

$$\alpha = \frac{R}{PV}$$

and isothermal compressibility:

$$\chi_T = \frac{RT}{VP^2}$$

where R is the ideal gas constant (Mayer's constant).

- Express the differential dV in terms of α, χ_T, dT, and dP.
- Through integration, deduce the equation of state of the gas, knowing that when $V = 2b$, $T = bP/R$. Recall the definitions:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P, \quad \chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

Exercise 3 (9 points)

Consider an elastic filament whose mechanical behavior is analogous to a spring. A thermodynamic state of this filament is defined by its temperature T and elongation l or tension force f, related by:

$$f = -alT^2$$

where a is a positive constant.

For an infinitesimal reversible transformation, the elementary work and heat received by the filament are:

$$\delta W = -f dl \quad \text{and} \quad \delta Q = C_l dT + \lambda dl$$

A. Determination of Calorimetric Coefficients and Entropy

1. Using the differential forms of the first and second laws of thermodynamics and the free energy, determine λ and $\left(\frac{\partial C_l}{\partial l}\right)_T$ in terms of T, l, and a.
2. Determine $C_l(l,T)$ knowing that at zero elongation the heat capacity is $C_l(0,T) = bT^2$ where b is a positive constant.
3. Determine the entropy state function and show that it can be expressed as:

$$S(l,T) = \frac{bT^2}{2} - al^2T + S_0$$

   where S_0 is a constant.

B. Isothermal Transformation

A mass m is suspended from the filament in Earth's gravitational field ($g = 9.81\,\mathrm{m\,s^{-2}}$). The experimenter slowly moves the mass from zero elongation (filament at rest) to the equilibrium position l_1. The transformation is reversible and occurs in contact with a thermostat at temperature T_0 (room temperature). Express results in terms of m, g, T_0, and a.

1. Determine l_1.
2. Calculate the work W_1 received by the filament.

0.2.10 Final Examination - Thermodynamics 10

Exercise 1 (5 points)

A gas with equation of state $V = V(T, P)$ has isobaric thermal expansion coefficient:

$$\alpha = \frac{R}{PV}$$

and isothermal compressibility:

$$\chi_T = \frac{RT}{VP^2}$$

where R is the ideal gas constant (Mayer's constant).

- Express the differential dV in terms of α, χ_T, dT, and dP.
- Through integration, deduce the equation of state of the gas, knowing that when $V = 2b$, $T = \frac{bP}{R}$. Recall the definitions:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P, \quad \chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

Exercise 2 (7 points)

One mole of monoatomic ideal gas contained in a cylinder undergoes a quasi-static and mechanically reversible cycle $ABCA$ as described. The process AB is isothermal at temperature $T_A = 301\,\mathrm{K}$. At point A, $P_A = 1.0\,\mathrm{bar}$. The process BC is isobaric at pressure $P_B = 5.0\,\mathrm{bar}$. The process CA is isochoric.

1. Calculate the volumes V_A, V_B, and V_C, and the temperature T_C.
2. Calculate the work and heat transfer received by the gas during each process AB, BC, and CA, and their sum.

Exercise 3 (8 points)

n moles of a monoatomic ideal gas undergo a cycle consisting of the following reversible transformations:

- An adiabatic process $A(P_A, V_A, T_2) \to B(P_B, V_B, T_1)$ with $T_1 > T_2$
- An isothermal expansion $B \to C(P_C, V_C, T_1)$
- An adiabatic process $C \to D(P_D, V_D, T_2)$
- An isothermal compression $D \to A$

Assume the heat capacity at constant volume is independent of temperature.

1. Sketch the cycle in the (P, V) plane (Clapeyron diagram).
2. Prove the relations $P_A P_C = P_B P_D$ and $V_A V_C = V_B V_D$.

3. Determine the work W_{AB}, W_{BC}, W_{CD}, and W_{DA} received by the gas in each process, expressed in terms of the initial and final state coordinates.
4. What is the relationship between W_{AB} and W_{CD}? Derive this directly using the First Law of Thermodynamics and the ideal gas properties.
5. Determine the heat transfers Q_{AB}, Q_{BC}, Q_{CD}, and Q_{DA} received by the gas during each process, expressed in terms of the cycle vertex coordinates, and specify their signs.
6. Establish a relationship between Q_{BC} and Q_{DA}.
7. Calculate the total work W received by the gas during the cycle.
8. Determine the cycle efficiency η.

Conclusion

The problems in this collection are organized into several core areas of thermodynamics:

1. **Equations of State and Thermodynamic Potentials**:
    - Exercises focus on deriving and applying equations of state for ideal and real gases (e.g., van der Waals equation)
    - Problems involving differential forms of internal energy (U), enthalpy (H), and entropy (S) emphasize the importance of exact differentials
2. **Heat Transfer and Thermal Properties**:
    - Questions on conduction, convection, and radiation explore practical applications
    - The use of coefficients like thermal expansion (α) highlights the interplay between properties
3. **Kinetic Theory and Statistical Mechanics**:
    - Problems involving Maxwellian velocity distributions bridge thermodynamics with statistical mechanics
    - The derivation of pressure as $p = \frac{1}{3}n_0 m v_q^2$ reinforces microscopic interpretation
4. **Thermodynamic Cycles and Irreversible Processes**:
    - Cycles are analyzed to calculate work, heat transfer, and efficiency (η)
    - Irreversible transformations illustrate key concepts like entropy production

This compilation serves as an effective tool for both learning and assessment:

- **Conceptual Understanding**: Tests foundational principles to advanced topics
- **Problem-Solving Skills**: Integrates mathematical rigor with physical intuition
- **Exam Preparation**: Mirrors typical exam formats with MCQs and structured problems

The problems reflect real-world challenges:

- **Energy Efficiency**: Questions on thermal resistance relevant to building insulation

- **Material Science**: Analysis of gas behavior for industrial gas storage
- **Environmental Physics**: Heat engines tie into sustainable energy conversion

This compilation successfully bridges theoretical thermodynamics with practical problem-solving. By mastering these exercises, learners will develop the analytical skills needed for scientific challenges.

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Exercise Collections and Problem Solving

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2. **J. P. Holman** - *Thermodynamics* (4th ed.), McGraw-Hill, 1988.
3. **Y. A. engel & M. A. Boles** - *Thermodynamics: An Engineering Approach* (9th ed.), McGraw-Hill, 2019.
4. **K. Wark & D. E. Richards** - *Thermodynamics* (6th ed.), McGraw-Hill, 1999.

French Thermodynamics References

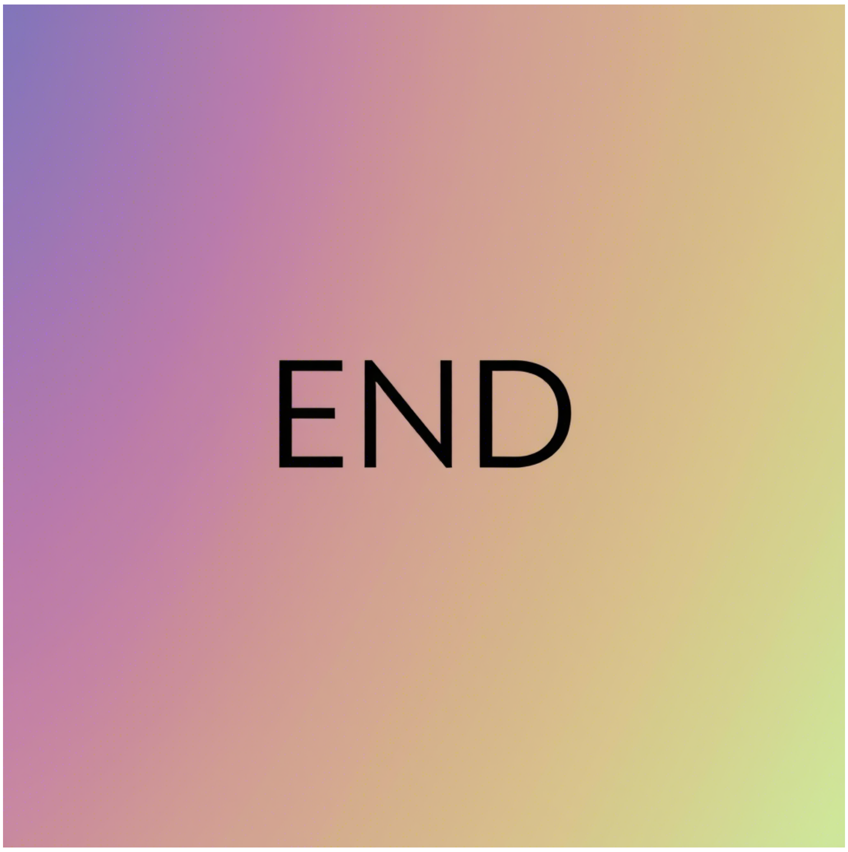
1. **J. H. Saac & C. G. Gueuret** - *Thermodynamique*, Dunod, 2005.
2. **F. L. R. Decamps** - *Problmes de Thermodynamique*, Masson, 1990.
3. **P. H. Lemarchand** - *Thermodynamique: Cours et Exercices*, Ellipses, 2002.

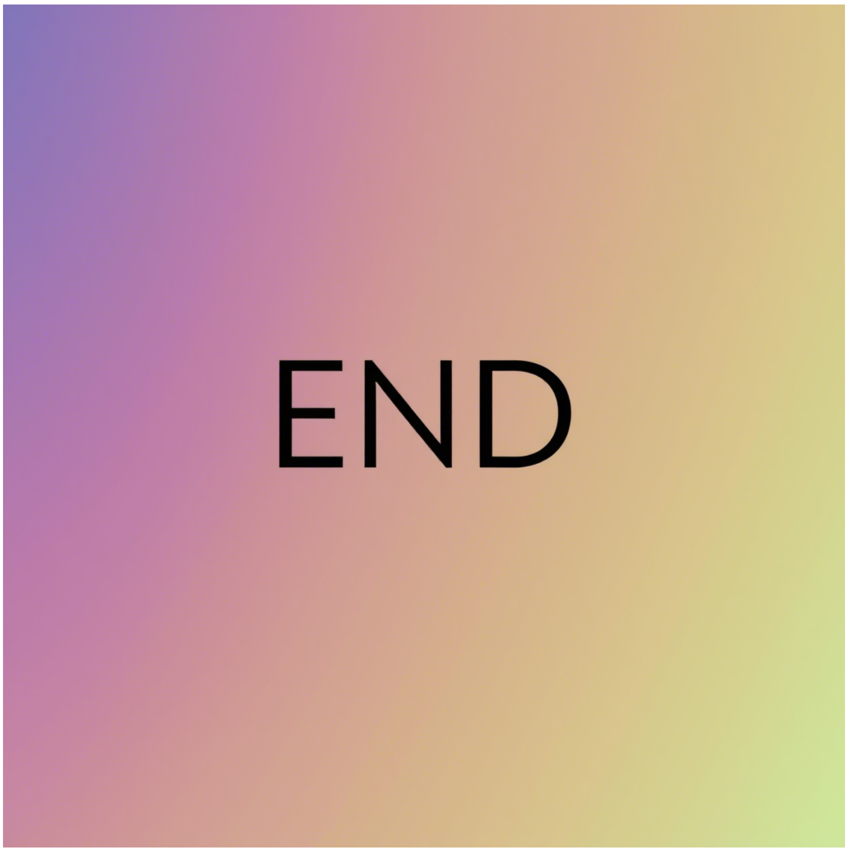
Advanced and Statistical Thermodynamics

1. **F. Reif** - *Fundamentals of Statistical and Thermal Physics*, McGraw-Hill, 1965.
2. **R. K. Pathria & P. D. Beale** - *Statistical Mechanics* (3rd ed.), Academic Press, 2011.
3. **K. Huang** - *Statistical Mechanics* (2nd ed.), Wiley, 1987.

Online Resources

1. MIT OpenCourseWare - Thermodynamics courses
2. Khan Academy - Thermodynamics modules
3. HyperPhysics - Thermodynamics concepts
4. Wolfram Demonstrations - Thermodynamic processes





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