Ibn Khaldoun University of Tiaret Faculty of Material Sciences
Department of Physics
First Year Bachelor's Degree in Physics

Exam maths 1 *18 /01 /2026* *120 mn*

Exercise 1 *4 pts* ***Choose the correct answer***

Let

$$A = [1, +\infty[\quad \text{and} \quad B = \left\{ \frac{1}{x} + 2, \quad 1 \leq x \leq 2 \right\}.$$

1. *We have*

a. $\inf A = 1$ *b.* $\inf A$ *does't exist* *c.* $\inf A = 2$

2. *We have*

a- $\max A = 1$ *b-* $\max A$ *does't exist* *c-* $\max A = +\infty$

3. *We have*

a- $\sup B = 3$ *b-* $\sup B$ *does't exist* *c-* $\sup B = \dfrac{5}{2}$

4- *let* $f(x) = \arcsin 2x$ *then the derivative of* f *is*

a. $f'(x) = \dfrac{2}{\sqrt{1-(2x)^2}}$ *b.* $f'(x) = \dfrac{2}{\sqrt{1+(2x)^2}}$ *c.* $f'(x) = \dfrac{1}{\sqrt{1-(2x)^2}}$

5- $\lim_{x \to 0} \frac{\sin 3x - \ln(1+5x)}{x}$ *equal*

a. 1 *b.* -2 *c.* 3

6- *If* $f(x) = \sqrt{\frac{x-1}{x^2+1}}$ *then the domain of defintion of* f *is*

a. $D_f = \mathbb{R} - \{-1\}$ *b.* $D_f = [1, +\infty[$ *c.* $D_f = \mathbb{R}^*$

7- *The integral of* $I = \displaystyle\int \frac{1}{1+x^2} dx$ *is*

a. $I = \arctan x + C$ *b.* $I = \tan x + C$ *c.* $I = \ln\left(x^2+1\right) + C$

8- *The integral of* $J = \displaystyle\int_0^1 \frac{1}{1+x} dx$ *is*

a. $J = \ln 2$ *b.* $J = 3$ *c.* $J = \dfrac{1}{2}$

Exercise 2 ***3 pts*** *Consider*

$$f(x)=\begin{cases}\sin x, & x<0,\\ e^{x}-e^{-x}, & x\geq 0.\end{cases}$$

1. Determine the domain of definition D_f
2. Study the continuity and derivability at $x_0=0$

Exercise 3 ***5 pts*** *Let*

$$I=\int_0^{\ln 2} x\mathbf{ch}^2 x dx \quad \text{and} \quad J=\int_0^{\ln 2} x\mathbf{sh}^2 x dx$$

where

$$\mathbf{ch}x=\frac{e^{x}+e^{-x}}{2} \quad \text{and } \mathbf{sh}x=\frac{e^{x}-e^{-x}}{2}$$

1 Show that

$$\mathbf{ch}^2 x-\mathbf{sh}^2 x=1 \text{ and } \mathbf{ch}^2 x+\mathbf{sh}^2 x=\mathbf{ch}2x$$

2- Detrmine by part the following integral

$$K=\int x\mathbf{ch}2x dx$$

3. Determine $I-J$ *and* $I+J$.
4. Deduce the value of I *and* J.

Exercise 4 ***5 pts*** *Let*

$$f(x)=\frac{1}{2x^2+x} \quad I=\int_1^2 f(x)\,dx \quad \text{and } J=\int_1^2 \frac{\ln(1+2t)}{t^2}dt.$$

1-Determine two real numbers a *and* b *such that, for* $x\in[1;2]$, *we have*

$$f(x)=\frac{a}{x}+\frac{b}{2x+1}.$$

2. Deduce the value of I
3. Determine by part the integral J

Exercise 5 ***3 pts*** *Let*

$$v_1(2,0,2),\ v_2(1,1,-1),\ v_3(-1,1,0) \ \text{ and } v(9,-3,0)$$

1- Show that v_1, v_2 *and* v_3 *are linear independant*
2- Show that v_1, v_2 *and* v_3 *are combination of* v.

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The model answer for the first exam maths 1 *18 /01 /2026* *120 mn*

1 Exercise 1 (4 points)

Exercise 1 ***4 pts Choose the correct answer***
Let

$$A = [1, +\infty[\quad and \quad B = \left\{ \frac{1}{x} + 2, \quad 1 \le x \le 2 \right\}.$$

1. *We have*

a. $\boldsymbol{x}$ $\inf A = 1$ *b.* $\inf A$ *does't exist* *c.* $\inf A = 2$

2. *We have*

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4- *let* $f(x) = \arcsin 2x$ *then the derivative of* f *is*

a. $\boldsymbol{x}$ $f'(x) = \frac{2}{\sqrt{1-(2x)^2}}$ *b.* $f'(x) = \frac{2}{\sqrt{1+(2x)^2}}$ *c.* $f'(x) = \frac{1}{\sqrt{1-(2x)^2}}$

5- $\lim_{x\to 0} \frac{\sin 3x - \ln(1+5x)}{x}$ *equal*

a. 1 *b.* $\boldsymbol{x}$ -2 *c.* 3

6- *If* $f(x) = \sqrt{\frac{x-1}{x^2+1}}$ *then the domain of defintion of* f *is*

a. $D_f = \mathbb{R} - \{-1\}$ *b.* $\boldsymbol{x}$ $D_f = [1, +\infty[$ *c.* $D_f = \mathbb{R}^*$

7- *The integral of* $I = \int \frac{1}{1+x^2} dx$ *is*

a. $\boldsymbol{x}$ $I = \arctan x + C$ *b.* $I = \tan x + C$ *c.* $I = \ln(x^2+1) + C$

8- *The integral of* $J = \int_0^1 \frac{1}{1+x} dx$ *is*

a. $\boldsymbol{x}$ $J = \ln 2$ *b.* $J = 3$ *c.* $J = \frac{1}{2}$

2 Exercise 2 (4 points)

Solution 2 *Consider the function defined by[cite: 55]:*

$$f(x) = \begin{cases} \sin x, & x < 0 \\ e^x - e^{-x}, & x \geq 0 \end{cases}$$

1. Determine the domain of definition D_f

The function is defined piecewise:

1- The domain of definition is the union of these intervals:

$$D_f =]-\infty, 0[\cup [0, +\infty[= \mathbb{R}.....0.5$$

2. Study the continuity and derivability at $x_0 = 0$

Continuity at $x_0 = 0$: For f to be continuous at 0, we must have $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$.

- Left limit: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sin x = 0$.
- Right limit: $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(e^x - e^{-x}) = e^0 - e^{-0} = 1 - 1 = 0$.
- Function value: $f(0) = e^0 - e^{-0} = 0$.

Since $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0) = 0$, the function is **continuous** at $x_0 = 0$....1 pt

Derivability at $x_0 = 0$: We calculate the left and right derivatives.

- **Left derivative ($f'_g(0)$):**

$$f'_g(0) = \lim_{x\to 0^-} \frac{f(x) - f(0)}{x} = \lim_{h\to 0^-} \frac{\sin x - 0}{x} = 1....0.5$$

- **Right derivative ($f'_d(0)$):**

$$f'_d(0) = \lim_{x\to 0^+} \frac{f(x) - f(0)}{x} = \lim_{x\to 0^+} \frac{(e^x - e^{-x}) - 0}{x} = \frac{0}{0} \text{ I F}$$

Using L'Hopital's rule:

$$\lim_{x\to 0^+} \frac{e^x - e^{-x}}{x} = \lim_{x\to 0^+} \frac{e^x + e^{-x}}{1} = 2....0.5 \text{ pts}$$

Since $f'_g(0) = 1$ and $f'_d(0) = 2$, we have $f'_g(0) \neq f'_d(0)$. Therefore, the function is **not differentiable** at $x_0 = 0$.....0.5 pts

Exercise 3 (5 points)

Solution 3 *Let* $I = \int_0^{\ln 2} x\mathbf{ch}^2 x\, dx$ *and* $J = \int_0^{\ln 2} x\mathbf{sh}^2 x\, dx$

1. Show the identities

(Assuming the question asks to verify these standard hyperbolic identities):

$$\mathbf{ch}x = \frac{e^x + e^{-x}}{2}, \quad \mathbf{sh}x = \frac{e^x - e^{-x}}{2}$$

$$\begin{aligned}
\mathbf{ch}^2 x - \mathbf{sh}^2 x &= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 \\
&= \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} \\
&= \frac{4}{4} = 1....0.5 \text{ pts}
\end{aligned}$$

$$\begin{aligned}
\mathbf{ch}^2 x + \mathbf{sh}^2 x &= \frac{e^{2x} + 2 + e^{-2x} + e^{2x} - 2 + e^{-2x}}{4} \\
&= \frac{2(e^{2x} + e^{-2x})}{4} = \frac{e^{2x} + e^{-2x}}{2} = \mathbf{ch}2x....0.5 \text{ pts}
\end{aligned}$$

2. Determine by parts the integral $K = \int x\mathbf{ch}2x\, dx$

Let $u = x \implies u' = 1$. Let $v' = \mathbf{ch}2x\, dx \implies v = \mathbf{sh}2x$.

Using integration by parts $\int udv = uv - \int vdu$:

$$\int x\mathbf{ch}2x\, dx = \frac{x}{2}\mathbf{sh}2x - \int \frac{1}{2}\mathbf{sh}2x\, dx$$

$$= \frac{x}{2}\mathbf{sh}2x - \frac{1}{2}\left(\frac{1}{2}\mathbf{ch}2x\right) + C$$

$$K(x) = \frac{x\mathbf{sh}2x}{2} - \frac{\mathbf{ch}2x}{4} + C....1 \text{ pt}$$

3. Determine $I - J$ and $I + J$

Calculate $I - J$:

$$I - J = \int_0^{\ln 2} x(\mathbf{ch}^2 x - \mathbf{sh}^2 x)\, dx$$

Since $\mathbf{ch}^2 x - \mathbf{sh}^2 x = 1$:

$$I - J = \int_0^{\ln 2} x\, dx = \left[\frac{x^2}{2}\right]_0^{\ln 2} = \frac{(\ln 2)^2}{2}....1 \text{ pt}$$

Calculate $I + J$:

$$I + J = \int_0^{\ln 2} x(\mathbf{ch}^2 x + \mathbf{sh}^2 x)\,dx = \int_0^{\ln 2} x\mathbf{ch}2x\,dx$$

Using the result $K(x)$ from question 2:

$$I + J = \left[\frac{x\mathbf{sh}2x}{2} - \frac{\mathbf{ch}2x}{4}\right]_0^{\ln 2}$$

We evaluate hyperbolic functions at $x = \ln 2$:

$$2x = 2\ln 2 = \ln 4$$

$$\mathbf{sh}(\ln 4) = \frac{e^{\ln 4} - e^{-\ln 4}}{2} = \frac{4 - \frac{1}{4}}{2} = \frac{\frac{15}{4}}{2} = \frac{15}{8}$$

$$\mathbf{ch}(\ln 4) = \frac{e^{\ln 4} + e^{-\ln 4}}{2} = \frac{4 + \frac{1}{4}}{2} = \frac{\frac{17}{4}}{2} = \frac{17}{8}$$

Now substitute:

$$I + J = \left(\frac{(\ln 2)(\frac{15}{8})}{2} - \frac{\frac{17}{8}}{4}\right) - \left(0 - \frac{1}{4}\right)$$

$$I + J = \frac{15\ln 2}{16} - \frac{17}{32} + \frac{8}{32} = \frac{15\ln 2}{16} - \frac{9}{32}....1 \text{ pt}$$

4. Deduce the value of I and J

We have the system: 1) $I - J = \frac{(\ln 2)^2}{2}$ 2) $I + J = \frac{15\ln 2}{16} - \frac{9}{32}$
Adding (1) and (2):

$$2I = \frac{(\ln 2)^2}{2} + \frac{15\ln 2}{16} - \frac{9}{32} \implies I = \frac{(\ln 2)^2}{4} + \frac{15\ln 2}{32} - \frac{9}{64}....0.5 \text{ pt}$$

Subtracting (1) from (2):

$$2J = \left(\frac{15\ln 2}{16} - \frac{9}{32}\right) - \frac{(\ln 2)^2}{2} \implies J = \frac{15\ln 2}{32} - \frac{9}{64} - \frac{(\ln 2)^2}{4}....0.5 \text{ pt}$$

Exercise 4 (5 points)

Solution 4 *Let* $f(x) = \frac{1}{2x^2+x}$ *and* $I = \int_1^2 f(x)dx$. *Let* $J = \int_1^2 \frac{\ln(1+2t)}{t^2}dt$.

1. Determine a and b

We want $f(x) = \frac{a}{x} + \frac{b}{2x+1}$.

$$\frac{1}{x(2x+1)} = \frac{a(2x+1)+bx}{x(2x+1)} = \frac{(2a+b)x+a}{x(2x+1)}$$

Comparing numerators: $a = 1$ and $2a + b = 0 \implies 2(1) + b = 0 \implies b = -2$.
Thus:

$$f(x) = \frac{1}{x} - \frac{2}{2x+1} \text{....1 pt}$$

2. Deduce the value of I

$$I = \int_1^2 \left(\frac{1}{x} - \frac{2}{2x+1}\right) dx$$

Recall that $\int \frac{u'}{u} = \ln|u|$.

$$I = [\ln|x| - \ln|2x+1|]_1^2$$

$$I = (\ln 2 - \ln 5) - (\ln 1 - \ln 3)$$

$$I = \ln 2 - \ln 5 - 0 + \ln 3 = \ln\left(\frac{2 \cdot 3}{5}\right) = \ln\left(\frac{6}{5}\right) \text{....2 pt}$$

3. Determine by parts the integral J

Let $u = \ln(1+2t) \implies u' = \frac{2}{1+2t}$. Let $v' = \frac{1}{t^2} \implies v = -\frac{1}{t}$.

$$J = \left[-\frac{\ln(1+2t)}{t}\right]_1^2 - \int_1^2 \left(-\frac{1}{t}\right) \frac{2}{1+2t} dt$$

$$J = \left(-\frac{\ln 5}{2}\right) - \left(-\frac{\ln 3}{1}\right) + 2\int_1^2 \frac{1}{t(1+2t)} dt$$

Notice the remaining integral is exactly I (with variable t instead of x):

$$\int_1^2 \frac{1}{2t^2+t} dt = I = \ln\left(\frac{6}{5}\right)$$

So:

$$J = -\frac{\ln 5}{2} + \ln 3 + 2(\ln 6 - \ln 5)$$

$$J = \ln 3 - \frac{\ln 5}{2} + 2\ln 6 - 2\ln 5 = \ln 3 + \ln 36 - \frac{5}{2}\ln 5$$

$$J_{int} = \ln(108) - \frac{5}{2}\ln 5 \text{....2 pt}$$

Exercise 5 (3 points)

Solution 5 *Let vectors:* $v_1(2,0,2)$, $v_2(1,1,-1)$, $v_3(-1,1,0)$ *and* $v(9,-3,0)$.

1. Show that v_1, v_2, v_3 are linearly independent

we have

$$\begin{aligned} \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 &= 0_{\mathbb{R}^3} \Rightarrow \begin{cases} 2\alpha_1 + \alpha_2 - \alpha_3 = 0 \\ 0\alpha_1 + \alpha_2 + \alpha_3 = 0 \\ 2\alpha_1 - \alpha_2 + 0\alpha_3 = 0 \end{cases} \\ &\Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0....1.5 \text{ pt} \end{aligned}$$

2. Show that v is a linear combination of v_1, v_2, v_3

v is a linear combination of v_1, v_2, v_3iff there exixt scalars α, β, γ such that

$$\alpha v_1 + \beta v_2 + \gamma v_3 = v$$

We solve for α, β, γ such that $\alpha v_1 + \beta v_2 + \gamma v_3 = v$:

$$\alpha \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 9 \\ -3 \\ 0 \end{pmatrix}$$

System of equations:

$$(S) \begin{cases} 2\alpha + \beta - \gamma = 9......(1) \\ \beta + \gamma = -3......(2) \\ 2\alpha - \beta = 0.......(3) \end{cases}$$

Substitute (3) into (2):

$$2\alpha + \gamma = -3 \implies \gamma = -3 - 2\alpha$$

Substitute β and γ into (1):

$$2\alpha + (2\alpha) - (-3 - 2\alpha) = 9$$

$$2\alpha + 2\alpha + 3 + 2\alpha = 9$$

$$6\alpha = 6 \implies \alpha = 1$$

Find β:

$$\beta = 2(1) = 2$$

Find γ:

$$\gamma = -3 - 2(1) = -5$$

Result: The vector v is a linear combination of the others:

$$v = 1v_1 + 2v_2 - 5v_3....1.5 \text{ pt}$$