University of Ibn-Khaldoun, Tiaret
Faculty of Matter Sciences

2sd year Physics
Exam: Crystallography, S3 (2025/26)

Exercise 01:
Draw a cubic unit cell and sketch the following crystallographic planes:
(110), (111), (210), $(11\bar{1})$, and (221).

Exercise 02:
An orthorhombic lattice has parameters:
a = 0.6 nm, b = 0.8 nm, and c =1.0nm.

(a) Derive the expressions for the reciprocal lattice vectors.
(b) Calculate their magnitudes.

Exercise 03:
Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be the primitive vectors of the unit cell.

Show that the scalar products between direct and reciprocal basis vectors satisfy:

$\vec{a}.\vec{b}^* = 0$ $\vec{a}.\vec{a}^* = 2\pi$

Exercise 04:
List and describe the symmetry elements that can exist in an orthorhombic unit cell. Illustrate each symmetry element with a simple schematic diagram.

Exercise 05:
Give the equivalent points obtained by the following symmetry operators:

- 2 // oz X (x y z)
- 2' (2 // oz and m in the xoy plane) X (r θ φ)

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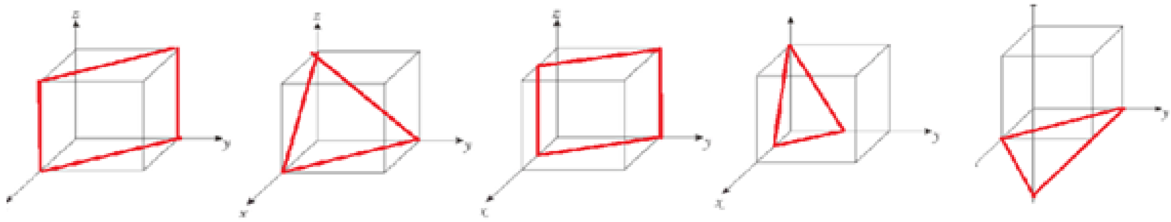
Exam correction: Crystallography, S3 (2025/26)

Exercise 01: (5.0pt)



(110) (1.0 pt) | (111) (1.0 pt) | (210) (1.0 pt) | (221) (1.0 pt) | $(11\bar{1})$ (1.0 pt)

Exercise 02: (4.5pt)

Orthorhombic lattice with parameters a = 0,6 nm, b = 0,8 nm et c = 1,0 nm.
Orthorhombic means the direct lattice vectors are mutually perpendicular:

$$\begin{cases} \vec{a} = a\,\vec{i} \\ \vec{b} = b\,\vec{j} \\ \vec{c} = c\,\vec{k} \end{cases}$$ (0.25 pt) + (0.25 pt) + (0.25 pt)

The expressions of the reciprocal lattice vectors are:

$$\begin{cases} \vec{a^*} = \frac{2\pi}{V}\left(b\,\vec{j} \wedge c\,\vec{k}\right) = \frac{2\pi}{a}\vec{i} & (0.25\ \text{pt}) + (0.5\ \text{pt}) \\ \vec{b^*} = \frac{2\pi}{V}\left(c\,\vec{k} \wedge a\,\vec{i}\right) = \frac{2\pi}{b}\vec{j} & (0.25\ \text{pt}) + (0.5\ \text{pt}) \\ \vec{c^*} = \frac{2\pi}{V}\left(b\,\vec{j} \wedge a\,\vec{i}\right) = \frac{2\pi}{c}\vec{k} & (0.25\ \text{pt}) + (0.5\ \text{pt}) \end{cases}$$

Where $V = \left|\vec{a}.\left(\vec{b} \wedge \vec{c}\right)\right| = a.b.c$ (0.25 pt) + (0.5 pt)

(b) Calculate their magnitudes

$$\begin{cases} \left|\vec{a^*}\right| = 10.47 \quad nm^{-1} & (0.25\ \text{pt}) \\ \left|\vec{b^*}\right| = 7.85 \quad nm^{-1} & (0.25\ \text{pt}) \\ \left|\vec{c^*}\right| = 6.28 \quad nm^{-1} & (0.25\ \text{pt}) \end{cases}$$

Exercise 03: **(3.0 pt)**

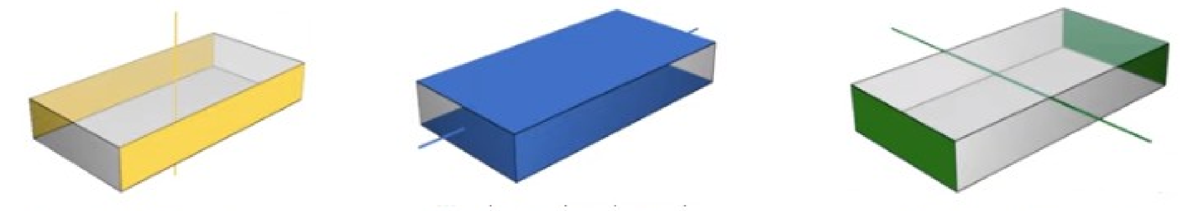
1- $\vec{\mathbf{b}}.\,\overrightarrow{\mathbf{a}^{*}} =?\ 0$

$$\vec{\mathbf{b}}.\left(\frac{2\pi}{\mathbf{v}}\left(\vec{\mathbf{b}}\wedge\vec{\mathbf{c}}\right)\right)=\frac{2\pi}{\mathbf{v}}\left(\vec{\mathbf{b}}.\left(\vec{\mathbf{b}}\wedge\vec{\mathbf{c}}\right)\right)=0 \quad (\vec{\mathbf{b}}\perp(\vec{\mathbf{b}}\wedge\vec{\mathbf{c}}))$$ (0.5 pt) + (1.0 pt)

2- $\vec{\mathbf{a}}.\,\overrightarrow{\mathbf{a}^{*}}=\vec{\mathbf{a}}.\left(\frac{2\pi}{\mathbf{v}}\left(\vec{\mathbf{b}}\wedge\vec{\mathbf{c}}\right)\right)=\frac{2\pi}{\mathbf{v}}\left(\vec{\mathbf{a}}.\left(\vec{\mathbf{b}}\wedge\vec{\mathbf{c}}\right)\right)=2\pi$ Since $\mathbf{v}=\left|\vec{\mathbf{a}}.\left(\vec{\mathbf{b}}\wedge\vec{\mathbf{c}}\right)\right|$ (0.5 pt) + (1.0 pt)

Exercise 04: **(4.5 pt)**

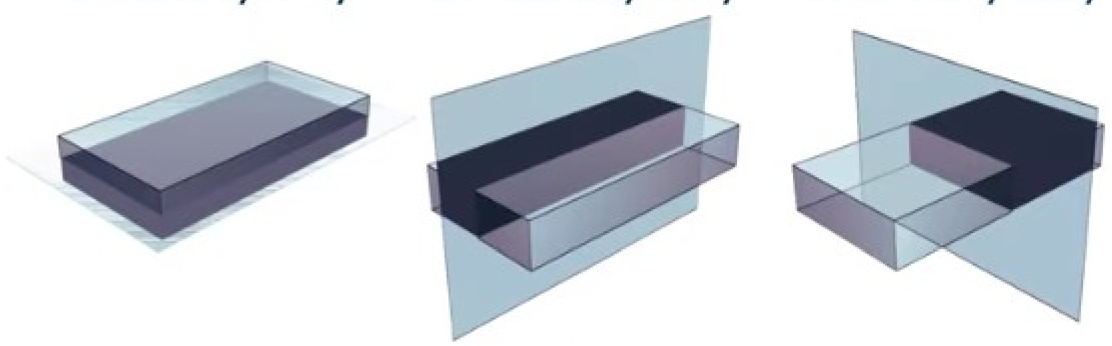
In the orthorhombic crystal system, the lattice parameters satisfy
$a \neq b \neq c$ (0.5 pt)
All inter axial angles are right angles:
$\alpha = \beta = \gamma = 90^{0}$ (0.5 pt)
An orthorhombic unit cell may contain the following symmetry elements:
(a) Three Mutually Perpendicular Twofold Rotation Axes ($3 \times C_2$) (0. 5 pt) + (0. 5 pt) + (0. 5 pt)
One twofold rotation axis along each crystallographic axis:
- C_{2x} along a $\vec{a}$, C_{2y} along $\vec{b}$ and C_{2z} along $\vec{c}$



1st Axis of Symmetry   2nd Axis of Symmetry   3rd Axis of Symmetry

(b) Mirror Planes (σ) (0. 5 pt) +(0. 5 pt) + (0. 5 pt)
- A mirror plane perpendicular to the a-axis (σ ⊥ a)
- A mirror plane perpendicular to the b-axis (σ ⊥ b)
- A mirror plane perpendicular to the c-axis (σ ⊥ c)

1st Plane of Symmetry   2nd Plane of Symmetry   3rd Plane of Symmetry



(c) Center of Inversion (i)   (0. 5 pt)

A point at the center of the unit cell

Exercise 05: **(3.0pt)**

The equivalent points obtained by the following symmetry operators:

- 2 // oz X (x y z): It is a second-order rotation about the Oz axis

The equivalent points are: $(\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ and $(\bar{\mathbf{X}}, \bar{\mathbf{Y}}, \mathbf{Z})$ (0.75 pt) + (0.75 pt)

- 2' (2 // oz and m in the xoy plane) X (r θ φ; It is a second -order rotation about the Oz axis followed by a reflection with respect to the xoy plane.

The equivalent points are: $(\mathbf{r}, \boldsymbol{\theta}, \boldsymbol{\varphi})$ and $(\mathbf{r}, \boldsymbol{\pi} - \boldsymbol{\theta}, \boldsymbol{\pi} + \boldsymbol{\varphi})$ (0. 75 pt) + (0.75 pt)